INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS B: ATOMIC, MOLECULAR AND OPTICAL PHYSICS

J. Phys. B: At. Mol. Opt. Phys. **39** (2006) S749–S766 doi:10.1088/0953-4075/39/15/S20

QED effects in a cavity with a time-dependent thin semiconductor slab excited by laser pulses

V V Dodonov[1] **and A V Dodonov**[2]

[1] Instituto de Física, Universidade de Brasília, Caixa Postal 04455, 70910-900 Brasília, DF, Brazil
[2] Departamento de Física, Universidade Federal de São Carlos, Via Washington Luiz Km 235, São Carlos, 13565-905, SP, Brazil

E-mail: vdodonov@fis.unb.br and adodonov@df.ufscar.br

Received 30 January 2006
Published 24 July 2006
Online at stacks.iop.org/JPhysB/39/S749

Abstract
We consider the problem of the creation of quanta of an electromagnetic field from the initial vacuum (or thermal) state in a closed high-Q cavity due to periodical variations of conductivity of a thin semiconductor boundary layer excited by short laser pulses. Fast changes of conductivity from practically a zero value to a high one and then again to zero simulate periodical displacements of the cavity wall. This scheme has been chosen to model the non-stationary Casimir effect in the experiment which is under preparation at Padua University. We provide analytical and numerical evaluations for the number of photons which could be created under realistic experimental conditions. We show the importance of taking into account intrinsic losses in the semiconductor slab caused by the finite conductivity during the intermediate part of the excitation–recombination cycle. We analyse the influence of different parameters, such as the diffusion and mobility coefficients of carriers, surface recombination velocity, absorption coefficient of laser radiation, thickness of the slab and geometry of the cavity. We conclude that a significant amount ($>10^3$) of 'Casimir photons' with a frequency of 2.5 GHz can be produced from vacuum in a cavity with dimensions of the order of 10 cm, if one can arrange several thousand strongly periodical laser pulses with a duration of the order of 1 ps, periodicity close to 200 ps and energy $\sim 10^{-3}$ J, illuminating the semiconductor slab of thickness ~ 1 mm and the mobility ~ 1 m^2 V^{-1} s^{-1}, provided the recombination time can be reduced below the critical value ~ 30 ps.

1. Introduction

Various quantum effects caused by the time dependence of the macroscopic dielectric properties of different media were the subjects of many publications over the past two decades. For example, the generation of squeezed states of electromagnetic field in time-dependent dielectrics without losses was considered in [1–6]. The photon generation from vacuum due to temporal variations of the dielectric function was discussed in [4, 7–14] and the generation of photons due to the motion of dielectric boundaries was studied in [15–22]. The phenomena of 'time refraction' were considered in [4, 5, 23]. The 'dynamical interferometers' with time-dependent reflection and transmission coefficients or spacetime field modulation were studied in [24, 25]). The generation of photons by time-varying beam-splitters was discussed in [26, 27].

Here we consider the problem of photon generation from vacuum (or initial thermal state) in the selected mode of electromagnetic field inside a closed high-Q cavity due to periodical variations of the conductivity of a thin semiconductor layer deposited on the plane surface of a cavity wall. Quantum effects caused by the time dependence of properties of thin slabs were studied by several authors [27–31]. However, only very simple models of the media were considered in those papers: lossless homogeneous dielectrics with time-dependent permeability [27, 31], ideal dielectrics or ideal conductors suddenly removed from the cavity [28, 29] or infinitely thin conducting slabs modelled by δ-potentials with time-dependent strength [30] (this model of 'plasma sheet' was introduced in [16]).

Fast and significant changes of electric properties can be achieved in semiconductors illuminated by laser pulses. The idea to use this scheme was put forward by Yablonovitch [7] who proposed to use a medium with a rapidly decreasing in time refractive index ('plasma window') to simulate the so-called Unruh effect. Man'ko [32] proposed to use semiconductors with time-dependent properties to produce the analogue of the *non-stationary Casimir effect* (see also [11, 33]). A more developed scheme, based on the creation of an electron–hole 'plasma mirror' inside a semiconductor slab, illuminated by a femtosecond laser pulse, was proposed in [34] (in the single-pulse case). But only recently a possibility of creating the effective 'plasma mirror' in a semiconductor slab was confirmed experimentally [35] (real plasma mirrors created by powerful ultrashort laser pulses were studied in [36]). Now a group in the University of Padua is preparing an experiment on observation of the non-stationary (dynamical) Casimir effect (NSCE) [37]. The idea is to use an effective electron–hole 'plasma mirror' created periodically on the surface of a semiconductor slab by illuminating it with a sequence of short laser pulses. If the interval between pulses exceeds the recombination time of carriers in the semiconductor, a highly conducting layer will periodically appear and disappear on the surface of the semiconductor film, thus simulating periodical displacements of the boundary. For cavities with oscillating ideal conducting boundaries, there exist theoretical predictions that under the condition of resonance between the frequency of the selected field mode and the frequency of boundary oscillations, the number of photons created from the vacuum state is given by the typical parametric resonance formula [38–43]

$$\mathcal{N}_n^{(\mathrm{res})} = \sinh^2(n\nu) \tag{1}$$

where n is the number of oscillations made by the wall and the small coefficient ν is proportional to the amplitude value of the relative shift of the mode eigenfrequency due to the change $\Delta L(t)$ of the distance L between the walls. Very roughly, $\nu \sim \max|\Delta L/L|$. Using a cavity of length $L \sim 10$ cm and semiconductor slabs of thickness $\sim$1 mm, one can expect to obtain a relative change of the field mode eigenfrequency inside the cavity of the order of 10^{-2}–10^{-3}. If formula (1) could be applied in this case, then one could expect that $\mathcal{N}_n$ would exceed the unit value and grow exponentially after $n > n_c \sim \nu^{-1} \sim 10^3$ oscillations.

However, the simple formula (1) does not take into account inevitable losses inside the semiconductor slab during the excitation–recombination process. This is the immediate consequence of the fact that the dielectric permeability $\epsilon(x)$ of the semiconductor medium is a *complex function*: $\epsilon = \epsilon_1 + \mathrm{i}\epsilon_2$, where $\epsilon_2 = 2\sigma/f_0$, σ and f_0 being the conductivity (in the

CGS units) and frequency in Hz, respectively. Good conductors have $\epsilon_2 \sim 10^8$ at microwave frequencies, which is essentially bigger than $\epsilon_1 \sim 1$–10. Although ϵ_2 is negligibly small in the non-excited semiconductor at low temperatures, it rapidly and continuously grows up to the values of the order of 10^5–10^6 during the laser pulse (for the maximum concentration of created carriers 10^{18}–10^{19} cm^{-3} and the mobility $b = 3 \times 10^6$ CGS units $= 1$ m^2 V s^{-1}), returning to zero after the recombination time.

Our goal is to give estimations of the number of photons which could be produced inside the cavity with a semiconductor time-dependent 'mirror', taking into account the internal dissipation in the slab, and to study the influence of different parameters on the rate of photon generation, such as the energy and periodicity of laser pulses, the bulk recombination time, surface recombination velocity, absorption coefficient of the laser radiation, mobility of carriers and the dimensions of the cavity. The case of ideal surfaces with zero surface recombination velocity was treated recently in [44–46]. Here we make the next step, considering a more realistic situation. This paper is organized as follows. In sections 2 and 3 we bring the necessary results of the papers [44–46]: the general formula for the number of photons which can be created from the initial thermal state in the selected field mode within the 'minimum noise' model of quantum damped oscillator with time-dependent parameters and a simple analytical interpolation formula for the complex frequency shift of the field mode due to the presence of a strongly inhomogeneous thin semiconductor slab with high conductivity nearby the surface and small conductivity in the rest part of the slab. The new results are contained in section 4, where we consider a concrete time dependence of the frequency shift and damping coefficient in the case of short laser pulses and analyse the influence of various parameters on the amount of created photons. Section 5 contains a brief discussion of the results obtained. The solution of the inhomogeneous diffusion equation for the spacetime distribution of the concentration of carriers inside the slab illuminated by short laser pulses is given in the appendix.

2. Photon generation rate in the presence of losses: general formulae

An immediate consequence of the time variation of the electromagnetic properties of the cavity walls is the time dependence of the eigenmode frequencies. Hence it follows a simple idea that one could understand the main features of the behaviour of the quantum field in the cavity by considering a single selected mode and describing it as a quantum oscillator with 'instantaneous' time-dependent frequency [47, 48]. Later on, it was justified (see, e.g., [38, 40, 49]) for three-dimensional cavities without accidental degeneracy of the spectrum of eigenmode frequencies and for harmonic variations of the effective frequency. We *assume* (although we have no strict mathematical proof) that even in the presence of dissipation and non-monochromatic periodical variations, the field problem still can be reduced approximately to the dynamics of the *single selected mode*, described in the classical limit as a harmonic oscillator with time-dependent *complex* frequency $\Omega(t) = \omega(t) - i\gamma(t)$ which can be found from the solution of the classical electrodynamical problem, taking the *instantaneous* geometry and material properties (as was done in the non-dissipative case in [31, 42]).

We use the model developed in [44, 45]. It can be considered as a generalization of the *quantum noise operator* approach, first proposed in [50] (for the recent applications of this approach see, e.g., [51]). It consists in the description of the dissipative quantum systems within the framework of the Heisenberg–Langevin operator equations. In the case concerned these equations can be written as

$$d\hat{x}/dt = \hat{p} - \gamma(t)\hat{x} + \hat{F}_x(t), \qquad d\hat{p}/dt = -\gamma(t)\hat{p} - \omega^2(t)\hat{x} + \hat{F}_p(t). \quad (2)$$

Here $\hat{x}$ and $\hat{p}$ are the dimensionless quadrature operators of the selected mode. We normalize these operators by the initial frequency in such a way that the mean number of photons equals $\mathcal{N} = \frac{1}{2}\langle \hat{p}^2 + \hat{x}^2 - 1\rangle$. In other words, in the subsequent formulae ω and γ are the frequency and damping coefficients normalized by the initial frequency ω_i. The noise operators $\hat{F}_x(t)$ and $\hat{F}_p(t)$ (which do not commute between themselves, but do commute with $\hat{x}$ and $\hat{p}$) are assumed to be delta-correlated:

$$\langle \hat{F}_j(t)\hat{F}_k(t')\rangle = \delta(t-t')\chi_{jk}(t), \qquad j,k = x,p, \tag{3}$$

$$\chi_{xp}(t) = -\chi_{px}(t) = \mathrm{i}\gamma(t), \qquad \chi_{pp}(t) = \chi_{xx}(t) = \gamma(t)G, \qquad G = 1 + 2\langle n\rangle_{\mathrm{th}}, \tag{4}$$

where $\langle n\rangle_{\mathrm{th}}$ is the equilibrium mean number of photons for the selected mode for the given temperature. The choice of coefficients in (2) and (4) was justified in [44].

The solution of the system of equations (2) under the condition (3) can be expressed in terms of the function $\varepsilon(t)$, which satisfies the *classical equation of motion* of the harmonic oscillator with time-dependent frequency

$$\ddot{\varepsilon} + \omega^2(t)\varepsilon = 0 \tag{5}$$

and the initial condition $\varepsilon(t) = \exp(-\mathrm{i}t)$ for $t \to -\infty$. Note that $\varepsilon(t)$ does not depend on the damping coefficient $\gamma(t)$. As was shown in [44], if initially (at $t \to -\infty$) the field mode was in the thermal state, then the mean number of photons at the instant t equals

$$\mathcal{N}(t) = G\,\mathrm{e}^{-2\Gamma(t)}\left\{\frac{1}{2}E(t) + \int_{-\infty}^{t}\mathrm{d}\tau\,\mathrm{e}^{2\Gamma(\tau)}\gamma(\tau)(E(t)E(\tau) - \mathrm{Re}[\tilde{E}^*(t)\tilde{E}(\tau)])\right\} - \frac{1}{2}, \tag{6}$$

where

$$\Gamma(t) = \int_{-\infty}^{t}\gamma(\tau)\,\mathrm{d}\tau, \qquad E(\tau) = \frac{1}{2}[|\varepsilon(\tau)|^2 + |\dot{\varepsilon}(\tau)|^2], \quad \tilde{E}(\tau) = \frac{1}{2}[\varepsilon^2(\tau) + \dot{\varepsilon}^2(\tau)]. \tag{7}$$

Formula (6) is *exact* for arbitrary functions $\omega(t)$ and $\gamma(t)$. However, we are interested here in the special case when the functions $\omega(t)$ and $\gamma(t)$ have the form of *periodical* pulses, separated by intervals of time with $\omega = 1$ and $\gamma = 0$ (we neglect the damping of the field between pulses, supposing that the quality factor of the cavity is big enough). Moreover, the relative change of the frequency $\omega(t)$ during pulses is very small. Under these conditions, the integral in (6) can be represented as a sum of integrals over subsequent pulses. This sum was calculated in the case of exact resonance in [44] and in a more general case (admitting the existence of some detuning from the exact resonance) in [46]. If $\varepsilon(t) = \mathrm{e}^{-\mathrm{i}t}$ before the 'effective potential barrier' $V_{\mathrm{eff}}(t) = \omega^2(t) - 1$, then after the 'barrier' the solution can be written as $\varepsilon(t) = \rho_-\,\mathrm{e}^{-\mathrm{i}t} + \rho_+\,\mathrm{e}^{\mathrm{i}t}$, so that the ratio $r = \rho_+/\rho_-$ can be interpreted as the effective complex amplitude reflection coefficient, whereas the complex number $\rho_- = f$ has the meaning of the inverse amplitude transmission coefficient. The mean number of created photons after n pulses can be expressed in terms of the coefficients r, f and the 'integral damping coefficient'

$$\Lambda = \int_{t_i}^{t_f}\gamma(\tau)\,\mathrm{d}\tau \tag{8}$$

(where t_i and t_f are the initial and final time moments of any pulse) as follows [46]:

$$\mathcal{N}_n = \frac{G|rf|^2\exp(-\Lambda)}{4\sinh(\nu)}\left[\frac{\exp[2n(\nu-\Lambda)]}{\sinh(\nu-\Lambda)} + \frac{\exp[-2n(\nu+\Lambda)]}{\sinh(\nu+\Lambda)}\right] + \frac{G-1}{2} - \frac{G|rf|^2[1+\exp(-2\Lambda)]}{4\sinh(\nu-\Lambda)\sinh(\nu+\Lambda)}. \tag{9}$$

The coefficient ν is determined by the relations

$$\cosh(\nu) = |\mathrm{Re}[f\exp(\mathrm{i}T)]| \equiv |f\cos(\delta)|, \qquad \delta = \omega_0(T - T_{\mathrm{res}}), \tag{10}$$

$$T_{\mathrm{res}} = \tfrac{1}{2}T_0(m - \varphi/\pi), \tag{11}$$

where $f = |f|\exp(\mathrm{i}\varphi)$, T is the periodicity of pulses, T_0 is the period of oscillations in the selected field mode and T_{res} is the resonance periodicity, when the effect is maximal. It is assumed in equation (9) that the coefficient ν is *real*. We see that photons can be generated provided $\nu > \Lambda$. Under the realistic experimental conditions the parameters $|r|$ and Λ are very small. Moreover, the parameter ν can be real only if the detuning coefficient δ is also small. Then one finds from (10) that $\nu \approx \sqrt{|r|^2 - \delta^2}$, so that no photons can be produced if the detuning coefficient exceeds the critical value $\delta_{\max} = \sqrt{|r|^2 - \Lambda^2}$.

Formula (9) can be simplified in the asymptotical regime $n(\nu - \Lambda) \gg 1$:

$$\mathcal{N}_n \approx \frac{G|r|^2}{4\nu(\nu - \Lambda)}\exp[2n(\nu - \Lambda)] + \frac{G-1}{2} \tag{12}$$

(we assume that the difference $\nu - \Lambda$ is not too close to zero). Thus one has to know only two parameters, $\nu \approx |r|$ and Λ, to calculate the mean number of photons in the resonance case.

For small variations of the effective frequency $\omega(t)$, we can write $\omega(t) = \omega_0[1 + \chi(t)]$ with $|\chi| \ll 1$. Then using the formulae from [52] we can express the absolute value of the single-barrier amplitude reflection coefficient $|r|$ and the phase φ of the single-barrier inverse transmission coefficient f as [53]

$$|r| \approx \left|\int_{t_i}^{t_f} \omega_0\chi(t)\,\mathrm{e}^{-2\mathrm{i}\omega_0 t}\,\mathrm{d}t\right|, \qquad \varphi \approx \omega_0\int_{t_i}^{t_f} \chi(t)\,\mathrm{d}t. \tag{13}$$

3. Frequency shift of the cavity mode caused by a thin inhomogeneous slab

In view of equation (13) we need the time dependence of the instantaneous eigenfrequency of the selected cavity mode (i.e., the eigenfrequency corresponding to the 'frozen' geometry of the cavity and distribution of the dielectric permeability at each instant of time). We consider a cylindrical cavity with an arbitrary cross section and the axis parallel to the x-direction, supposing that the main part of the cavity is empty, except for a thin slab of a semiconductor material. Thus we write $\varepsilon(x) \equiv 1$ for $-L < x < 0$ and $\varepsilon(x) \neq 1$ for $0 < x < D$, where D is the thickness of the slab and L is the cavity length. We assume that $D \ll L$ and the dielectric permeability depends only on the longitudinal space variable x.

The set of Maxwell's equations can easily be reduced to the following second-order partial differential equation for the monochromatic component of the electric induction vector $\mathbf{D}(\mathbf{r}, t) = \mathbf{D}(\mathbf{r})\exp(-\mathrm{i}\Omega t)$:

$$\mathrm{graddiv}(\mathbf{D}/\varepsilon) - \Delta(\mathbf{D}/\varepsilon) = (\Omega^2/c^2)\mathbf{D}. \tag{14}$$

The use of vector $\mathbf{D}$ is explained by the fact that in such a case Maxwell's equations do not contain derivatives of function $\varepsilon(\mathbf{r}, t)$ with respect to the time variable. Solving equation (14), containing time variable t as a parameter through the function $\varepsilon(\mathbf{r}, t)$, one can find the time-dependent instantaneous eigenfrequency $\Omega(t)$. For the TE modes, vectors $\mathbf{E}$ and $\mathbf{D}$ are perpendicular to the x axis (and parallel to plane surfaces of the cylinder and slab). Then the electric field $\mathbf{E} = \mathbf{D}/\varepsilon(x)$ satisfies the usual three-dimensional Helmholtz equation $\Delta\mathbf{E} + (\Omega/c)^2\varepsilon(x)\mathbf{E} = 0$, which allows one to factorize any scalar component of the electric field as $E(x, \mathbf{r}_\perp) = \psi(x)\Phi(\mathbf{r}_\perp)$, where the function $\Phi(\mathbf{r}_\perp)$ obeys the two-dimensional Helmholtz equation $\Delta_\perp \Phi + k_\perp^2 \Phi = 0$. Consequently, the problem is reduced to solving the one-dimensional Helmholtz equation

$$\psi'' + \left[(\Omega/c)^2 \varepsilon(x) - k_\perp^2\right] \psi = 0. \tag{15}$$

Its solution in the domain $-L < x < 0$ satisfying the boundary condition $\psi(-L) = 0$ is $\psi(x) = F_1 \sin[k(x+L)]$, where the constant coefficient k is related to the field eigenfrequency Ω and the corresponding wavelength in vacuum λ as

$$\Omega = c\left(k^2 + k_\perp^2\right)^{1/2}, \qquad \lambda = 2\pi\left(k^2 + k_\perp^2\right)^{-1/2}. \tag{16}$$

The conditions of continuity of the function $\psi(x)$ and its derivative at $x = 0$ result in the transcendental equation for the wave number k

$$\tan(kL) = k\psi_+(0; k)/\psi'_+(0; k), \tag{17}$$

where $\psi_+(x; k)$ is the solution of equation (15) in the domain $0 < x < D$ satisfying the boundary condition $\psi_+(D) = 0$. In the case of thin slab with $D \ll \lambda \sim L$, the value of k must be close to π/L (we consider the lowest mode of the cavity). Thus we can write $k = (1+\xi)\pi/L$ with $|\xi| \ll 1$ and replace $\tan(\pi\xi)$ on the left-hand side of (17) simply by $\pi\xi$. Moreover, with the same accuracy we can identify k with π/L on the right-hand side. Thus we arrive at the formula $\xi = \eta\Delta R(0)$, where the function $R(\tilde{x}) = \tilde{\psi}_+(\tilde{x})/\tilde{\psi}'_+(\tilde{x})$ of the dimensionless variable $\tilde{x} = x/D$ satisfies the first-order nonlinear *generalized Riccati equation*

$$\frac{\mathrm{d}R}{\mathrm{d}\tilde{x}} = 1 + \pi^2\Delta^2[\varepsilon(\tilde{x}) - 1 + \eta^2]R^2 \tag{18}$$

in the domain $0 \leqslant \tilde{x} \leqslant 1$ and the boundary condition $R(1) = 0$. The dimensionless parameters η and Δ are defined as

$$\eta = \frac{\lambda}{2L} < 1, \qquad \Delta = \frac{2D}{\lambda} \ll 1. \tag{19}$$

The small relative shift of the resonance frequency can be expressed as

$$\chi_\Omega \equiv [\Omega - \omega_0]/\omega_0 = \eta^2(\xi - \xi_0) = \eta^3\Delta[R(0) - R_0(0)],$$

where ξ_0 or $R_0(0)$ correspond to the non-excited semiconductor slab with $\varepsilon(\tilde{x}) = \varepsilon_1 = \text{const}$. In this case, equation (18) has exact solution which shows that, for $\varepsilon_1 \sim 10$ (typical values for semiconductors), $R_0(0) \approx -1$ with an accuracy of the order of 0.01 or even better. When the semiconductor slab is illuminated by the laser pulse, $\varepsilon(\tilde{x}) = \varepsilon_1 + \mathrm{i}\varepsilon_2(\tilde{x})$, where the imaginary part $\varepsilon_2(\tilde{x})$ can attain very big values, so that $\pi^2\Delta^2\varepsilon_2(\tilde{x}) \gg 1$ in some region $0 < \tilde{x} < \tilde{x}_0$ near the surface of the slab. Obviously, to create an effective 'plasma mirror' one needs the material with $\tilde{x}_0 \sim (\alpha D)^{-1} \ll 1$, where α is the absorption coefficient of the laser radiation. It is important that the generation of new carriers by the laser pulse influences the imaginary part ε_2 of the dielectric function but it practically does not change the real part ε_1 at the frequency of the fundamental mode (~5 GHz), because this frequency is much less than the optical frequencies and the concentration of new carriers is still much less than the concentration of 'optical electrons' in the material (i.e., the corresponding plasma frequency is much less than the frequency of optical transitions).

In the region $0 < \tilde{x} < \tilde{x}_0$ we can neglect the first term 1 on the right-hand side of equation (18), as well as the real part of the coefficient at R^2. The simplified equation can be integrated immediately, resulting in the formula

$$\frac{1}{R(0)} - \frac{1}{R(\tilde{x}_0)} = \mathrm{i}(\pi\Delta)^2 \int_0^{\tilde{x}_0} \varepsilon_2(\tilde{x})\,\mathrm{d}\tilde{x}. \tag{20}$$

On the other hand, in the region $\tilde{x}_0 < \tilde{x} < 1$ the nonlinear term in (18) becomes insignificant, so that we can write $R(1) - R(\tilde{x}_0) = 1 - \tilde{x}_0$. Since $R(1) = 0$ and $\tilde{x}_0 \ll 1$, we can take $R(\tilde{x}_0) = -1$. Moreover, since the function $\varepsilon_2(\tilde{x})$ quickly goes to zero outside the interval $(0, \tilde{x}_0)$, we can extend the upper limit of integration in (20) to infinity. Thus we have $R(0) = (\mathrm{i}A - 1)^{-1}$ and finally [45]

$$\chi_\Omega = \eta^3 \Delta \frac{A^2 - \mathrm{i}A}{A^2 + 1}, \qquad \chi = \eta^3 \Delta \frac{A^2}{A^2 + 1}, \qquad \gamma = \eta^3 \Delta \frac{A}{A^2 + 1} \tag{21}$$

where

$$A = (\pi \Delta)^2 \int_0^\infty \varepsilon_2(\tilde{x})\, \mathrm{d}\tilde{x}. \tag{22}$$

Consequently, we need only the integral of the instantaneous concentration of the carriers over the slab at every instant of time t to find the functions $\chi(t)$ and $\gamma(t)$. Obviously, the approximate formulae in (21) are not very good if $A \ll 1$. But the contributions of the corresponding pieces of the functions $\chi(t)$ and $\gamma(t)$ to the integrals (8) and (13) are so small that they can be neglected. We have inspected the solution (21) by solving equation (18) numerically for two dielectric functions $\varepsilon(\tilde{x}) = 10 + \mathrm{i}B \exp(-\tilde{\alpha}\tilde{x})$ (in this case equation (15) can be solved analytically in terms of Bessel functions [53]) and $\varepsilon(\tilde{x}) = 10 + \mathrm{i}B \cosh^{-2}(\tilde{\alpha}\tilde{x})$ with different parameters B and $\tilde{\alpha}$. The coefficient A (22) in both cases equals $B/\tilde{\alpha}$. For $\tilde{\alpha} = 100$ the relative difference between the numerical solutions and analytical expressions in (21) was less than 0.001 for any value $B > 100$ (for both functions). The difference at the level of a few per cent was noted only for values $\tilde{\alpha} < 10$. These results show that the approximation (21) is quite adequate under the conditions concerned.

Equation (21) shows that big values of χ can be achieved asymptotically for $A_0 \gg 1$. On the other hand, the maximal value of γ is achieved for $A_0 = 1$, and this value is only twice smaller than the maximal value of χ. This means that the influence of damping cannot be neglected if one wants to have big values of the relative frequency shift.

Surprisingly, in the case of TM modes the functions χ and γ are given by the same relations (21), with the only difference that the factor η^3 should be replaced by η in the first power [46]. This means that in the cavities with *the same* geometry the rate of generation is higher for the TM modes than for TE ones [31, 41, 53]. However, having in mind the applications to the real experimental conditions [37], where the mode TE_{101} in a rectangular cavity is used, we shall perform all further evaluations only for this special case (because the mode TM_{101} does not exist for this geometry).

4. Influence of the slab properties on the photon generation rate

We see that the instantaneous shift of frequency and damping coefficient are determined by the integral of the imaginary part ε_2 of the dielectric function over the slab. Due to the relations $\varepsilon_2 = 2\sigma/f_0$ and $\sigma = n|eb|$, we can rewrite (22) as

$$A(t) = (4\pi^2 \Delta |eb|/c)\mathcal{K}(t) \tag{23}$$

where

$$\mathcal{K}(t) = \int_0^\infty n(x, t)\, \mathrm{d}x \tag{24}$$

can be interpreted as the 'effective surface concentration' of electron–hole pairs created inside the slab, n is the volume concentration of these pairs, e is the electron charge, $c = f_0\lambda$ is the velocity of light and b is the total mobility of carriers for each electron–hole pair (we use the CGS units). We assume that the mobility does not depend on the position (in particular, that it does not depend on concentration). The extension of the limit of integration to infinity is justified for the absorption length α^{-1} much smaller than the thickness of the slab D.

The dependence $n(x, t)$ can be found from equations which take into account, besides the photo-absorption, the effect of diffusion and different recombination processes. We use the simplified version of the equation considered in [54–57]

$$\partial n/\partial t = \nabla \cdot (Y \nabla n) + (\alpha\zeta/E_s) I(t)\, e^{-\alpha x} - \beta_1 n. \tag{25}$$

Here Y is the coefficient of ambipolar diffusion, α is the absorption coefficient of the laser radiation inside the layer, E_s is the energy gap of the semiconductor (which is close to the energy of laser photons), $I(t)$ is the time-dependent intensity of the laser pulse which enters the slab (it can be less than the intensity of the pulse outside the slab, because the reflection coefficient from the semiconductor surface can be rather big, due to the big value of the dielectric constant $\varepsilon_1 \sim 10$; however, the reflection can be diminished if some quarter-wavelength film is put on the surface), $\zeta \leqslant 1$ is the efficiency of the photo-electron conversion and β_1 is the trap-assisted recombination coefficient.

The simplification made consists in omitting on the right-hand side of (25) the terms $-\beta_3 n^3 - \beta_2 n^2$, where β_3 is the Auger recombination coefficient and β_2 is the radiative recombination coefficient. With these nonlinear terms, equation (25) can be solved only numerically [55–57]. However, for modelling the non-stationary Casimir effect, one needs very small recombination times, of the order of $T_r \sim 20$–30 ps (see the evaluations at the end of this section). This can be achieved in doped materials with high concentration of impurities. The doping increases the coefficient β_1 but does not change β_2 and β_3 significantly. Thus we can assume that $\beta_1 \approx T_r^{-1} \sim 5\times 10^{10}$ s^{-1}. On the other hand, the typical values of coefficients β_2 and β_3 are of the order of [55–57] $\beta_2 \sim 5 \times 10^{-14}$ cm^3 s^{-1} and $\beta_3 \sim 4 \times 10^{-31}$ cm^6 s^{-1} (these are the data for Si, but we suppose that the orders of magnitude in GaAs are the same). Then even for $n \sim 10^{18}$ cm^{-3} we have $\beta_2 n \sim 5 \times 10^4$ s^{-1} and $\beta_3 n^2 \sim 4 \times 10^5$ s^{-1}. These values are six and five orders of magnitude smaller than β_1. Consequently, in the special case of highly doped materials, which corresponds to the conditions of the proposed experiment, the nonlinear recombination terms can be neglected without any doubts.

For powerful pulses one should add the equation for the spacetime evolution of temperature, because many coefficients in equation (25) are temperature-sensitive (moreover, this equation contains in the generic case the term proportional to the temperature gradient [56]). We suppose that the change of temperature and its gradients are small, so that the coefficients do not depend on time (and on position, too). In the most general case, one should use the function $I(t - x/v)$ instead of $I(t)$, where v is the group velocity. But for materials with high absorption coefficients the coordinate dependence can be frequently neglected. For example, if $\alpha = 10^5$ cm^{-1} and $v \sim 10^{10}$ cm s^{-1}, then $x/v < x_0/v \sim (\alpha v)^{-1} \sim 10^{-15}$ s, which is much less than the characteristic time of the laser pulse. We suppose that the duration of each pulse is of the order of a few picoseconds. Then equation (25) is justified if $\alpha > 10^2$ cm^{-1}. (Note that smaller values of α would result in the non-total absorption of the laser radiation inside the slab, so that materials with $\alpha < 10^2$ cm^{-1} should hardly be used).

The boundary condition to equation (25) is [55–57]

$$Y \left.\frac{\partial n}{\partial x}\right|_{x=0} = R n(0) \tag{26}$$

where R is the surface recombination velocity [58]. Since equation (25) is *linear*, it can be solved analytically (see the appendix). If $n(x, t) = 0$ in the absence of the inhomogeneous term $I(t)$, then (for $Y =$ const)

$$n(x, t) = \frac{\zeta\alpha}{E_s} \int_0^\infty dt' \exp(-\beta_1 t') I(t - t') \Psi_g(x, t') \tag{27}$$

where

$$\Psi_g(x,t') = \frac{1}{\pi}\,\mathrm{Re}\left[\int_0^\infty \mathrm{d}\kappa\,\exp(\mathrm{i}\kappa\,\alpha x - \kappa^2\alpha^2 Y t')\left(\frac{1}{1+\mathrm{i}\kappa} + \frac{1}{1-\mathrm{i}\kappa}\,\frac{\kappa-\mathrm{i}g}{\kappa+\mathrm{i}g}\right)\right], \tag{28}$$

$$g = R/(\alpha Y). \tag{29}$$

We assume that $\alpha D \gg 1$, so that the presence of the right boundary of the semiconductor slab does not affect the carrier distribution near the irradiated surface.

Integrating both sides of equation (24) over x from 0 to ∞ taking into account equation (26) for the derivative $\partial n/\partial x$ at $x = 0$ (arising from the diffusion term) and formulae (27) and (28) (with $x = 0$) for the term $n(0)$, we arrive at the closed inhomogeneous ordinary differential equation of the first order

$$\frac{\mathrm{d}\mathcal{K}}{\mathrm{d}t} = -\beta_1\mathcal{K} + \frac{\zeta}{E_\mathrm{s}}[I(t) + \tilde{I}(t)], \tag{30}$$

where

$$\tilde{I}(t) = \frac{2}{\pi} R\alpha(1+g)\int_0^\infty \mathrm{d}t'\,\exp(-\beta_1 t')I(t-t')\int_0^\infty \mathrm{d}\kappa\,\frac{\kappa^2\exp(-\kappa^2\alpha^2 Y t')}{(1+\kappa^2)(\kappa^2+g^2\Delta)}. \tag{31}$$

Equation (30) can be integrated immediately (we suppose that the laser pulse starts at $t = 0$, so that $I(t) = \mathcal{K}(t) = 0$ for $t < 0$):

$$\mathcal{K}(t) = (\zeta/E_\mathrm{s})\int_0^t \exp[-\beta_1(t-\tau)][I(\tau) + \tilde{I}(\tau)]\,\mathrm{d}\tau. \tag{32}$$

Supposing that the duration of laser pulse is much less than the recombination time, we approximate the function $I(t)$ by the delta-function $I(t) = (W/S)\delta(t)$, where W is the total energy of the laser pulse and S is the area of the surface of the semiconductor slab (we assume that the energy is distributed uniformly over this area). Then we obtain the following expression for the time-dependent function $A(t)$ in equation (21) (strictly speaking, we believe the argument of the delta-function to be slightly shifted from 0, so that the lower limit of integration in (32) does not influence the result):

$$A(\tau) = A_0\,\mathrm{e}^{-\tau/Z}\mathcal{J}_g(\tau,h) \tag{33}$$

where

$$\mathcal{J}_g(\tau,h) = 1 + \frac{2}{\pi}g(1+g)\int_0^\infty \frac{\mathrm{d}\kappa[1-\exp(-h\kappa^2\tau)]}{(1+\kappa^2)(\kappa^2+g^2)} \tag{34}$$

and the new dimensionless variables and parameters are defined as follows:

$$\tau = \omega_0 t \qquad Z = \frac{\omega_0}{\beta_1} = \frac{2\pi T_\mathrm{r}}{T_0} \qquad A_0 = \frac{4\pi^2|eb|\zeta W\Delta}{(cE_\mathrm{s}S)} \qquad h = \frac{\alpha^2 Y}{\omega_0}. \tag{35}$$

The integral in (34) can be expressed in terms of the complementary error function

$$\mathrm{Erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-t^2)\,\mathrm{d}t \tag{36}$$

due to formula (2.34) from [59]

$$\int_0^\infty \frac{\mathrm{d}t\,\exp(-pt)}{\sqrt{t}(a+t)} = \frac{\pi}{\sqrt{a}}\,\mathrm{e}^{ap}\,\mathrm{Erfc}(\sqrt{ap}). \tag{37}$$

Thus we have

$$\mathcal{J}_g(\tau,h) = \frac{1}{g-1}[g\,\mathrm{e}^{h\tau}\,\mathrm{Erfc}(\sqrt{h\tau}) - \mathrm{e}^{h\tau g^2}\,\mathrm{Erfc}(g\sqrt{h\tau})]. \tag{38}$$

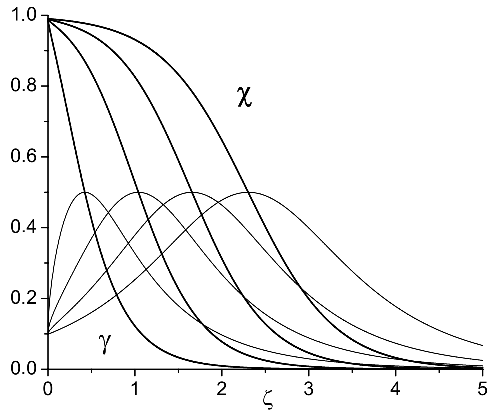

Figure 1. The relative shift of the cavity eigenfrequency χ (upper curves) and normalized damping coefficient γ (lower curves) versus the dimensionless time variable $\zeta = \beta_1 t = \tau/Z$ for $A_0 = 10$ and fixed values of the parameters g and h (from right to left): $g = 0$ and h arbitrary; $g = 10$ and $h = 1$; $g = 10$ and $h = 10$; $g = 10$ and $h = 100$.

If $g = 0$ (an ideal surface with zero surface recombination velocity), then $\mathcal{J}_0(\tau, h) \equiv 1$, so that the function $A(\tau)$ is reduced to a simple exponential function independently of the values of the diffusion and absorption coefficients. This special case was studied in [45]. Also $\mathcal{J}_g(0, h) \equiv 1$ for any g and h. For $g = 1$ formula (38) can be written as

$$\mathcal{J}_1(\tau, h) = (1 - 2h\tau)\,\mathrm{e}^{h\tau}\,\mathrm{Erfc}(\sqrt{h\tau}) + 2\sqrt{h\tau/\pi}$$

and for $g = \infty$ we obtain

$$\mathcal{J}_\infty(\tau, h) = \mathrm{e}^{h\tau}\,\mathrm{Erfc}(\sqrt{h\tau}).$$

For $h\tau \ll 1$ and $g^2 h\tau \ll 1$ we have $\mathcal{J}_g(\tau, h) \approx 1 - gh\tau$, whereas for $h\tau \gg 1$ and $g^2 h\tau \gg 1$ we have $\mathcal{J}_g(\tau, h) \approx (g+1)/\sqrt{\pi g^2 h\tau}$. (Remember that $\mathrm{Erfc}(x) \approx (\sqrt{\pi}x)^{-1}\exp(-x^2)$ for $x \gg 1$.) Consequently, the surface recombination causes a faster return of the functions $\chi(\tau)$ and $\gamma(\tau)$ to zero values after the excitation. This is shown in figure 1.[3]

We see that the integral damping coefficient Λ (which is proportional to the area under the curve $\gamma(\tau)$) decreases with increase of h for big values of g (high surface recombination velocity). But the effective reflection coefficient from the time-dependent 'potential barrier' $\chi(\tau)$ also diminishes, because the 'barrier' becomes more narrow preserving its height. According to equation (12), the rate of the photon generation is determined by the 'competition' between these two effects, namely by the difference

$$\nu - \Lambda = \eta^3 \Delta F_g(A_0, Z, h), \qquad F_g(A_0, Z, h) = \tilde{\nu} - \tilde{\Lambda} \tag{39}$$

where the coefficients $\tilde{\nu}$ and $\tilde{\Lambda}$ can be expressed due to equations (8), (13), (21) and (33) as

$$\tilde{\nu} = Z\left|\int_0^\infty \mathrm{d}x\,\frac{[A_0\mathcal{J}_g(x)\exp(-x[1+\mathrm{i}Z])]^2}{1 + [A_0\mathcal{J}_g(x)\exp(-x)]^2}\right|, \tag{40}$$

[3] As a matter of fact, the function $\chi(\tau)$ rapidly increases from zero at $\tau = 0 - \epsilon$ (where ϵ is the dimensionless duration of the laser pulse) to almost unit value at $\tau = 0$. The function $\gamma(\tau)$ also starts from zero at $\tau = 0 - \epsilon$ and shows a very narrow splash of the height 0.5 before reaching the apparent initial value 0.1 at $\tau = 0$ shown in the figure. However, these details cannot be seen in the plots in the case $\epsilon \to 0$ considered here (a very short laser pulse), and they do not influence the values ν and Λ in this limit.

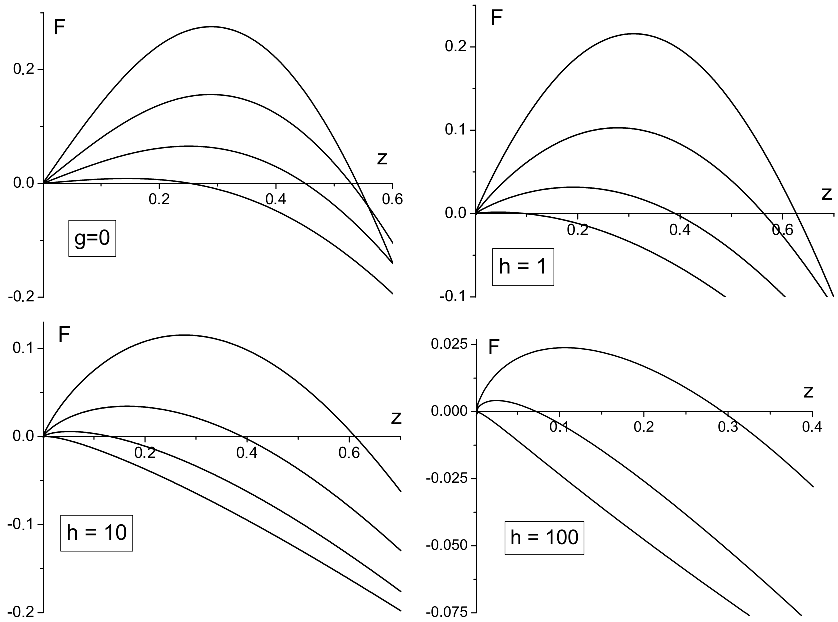

Figure 2. The dependence of the amplification coefficient F on the parameter Z for fixed values of the parameter $A_0 = 4, 6, 10, 20$ (from bottom to top; in the case of $h = 100$ the plots are made for $A_0 = 4, 10, 20$). The plots with $g = 0$ do not depend on h. In all plots with fixed values of h we use the value $g = 10$.

$$\tilde{\Lambda} = Z \int_0^\infty \mathrm{d}x \frac{A_0 \mathcal{J}_g(x) \exp(-x)}{1 + [A_0 \mathcal{J}_g(x) \exp(-x)]^2}. \tag{41}$$

The case of $g = 0$ (when the dependence on parameter h disappears) was studied in [45]. Here we study the influence of parameters g and h. In figure 2 we show the dependence $F(Z)$ for different fixed values of g, h and A_0. Since $\tilde{\Lambda}$ is proportional to Z due to formula (41), the amplification factor F inevitably becomes negative for big values of Z, because the 'effective reflection coefficient' from a single barrier $\tilde{\nu}$ is obviously limited from above. Consequently, one needs materials with small values of Z, i.e., the recombination time.

In figure 3 we show the dependence $F(h)$ for the fixed values $A_0 = 10$ and $Z = 0.3$ (which are optimal in the case $g = 0$ [45]) and with different values of g. We see that for nonzero surface recombination velocity the increase of h (in particular, the increase of the absorption coefficient) diminishes the value of F. Moreover, figure 2 shows that the increase of h diminishes the maximal possible values of the recombination time (parameter Z) for which $F > 0$ (if $h \gg 1$).

Usual mechanically polished semiconductor surfaces have the surface recombination velocity $R \sim 10^6$–10^7 cm s^{-1} [58]. Using the Einstein relation $Y = k_B T b/|e|$ we estimate the diffusion coefficient at $T = 4$ K as $Y \sim 3$ cm^2 s^{-2} for $b \sim 10^4$ cm^2 V s^{-1}. Thus for usual surfaces the parameter g is greater than unity; moreover, it can be much greater than unity for the values of absorption coefficient smaller than 10^6 cm^{-1}. Figures 2 and 3 show that in order to have sufficiently big values of the amplification factor F, the parameter h must be smaller than unity (by the order of magnitude). For the frequency 2.5 GHz this means that the absorption coefficient α should not exceed the value of the order of 10^5 cm^{-1}. The physical reason for the limitation of h from above seems to be clear. The initial thickness of

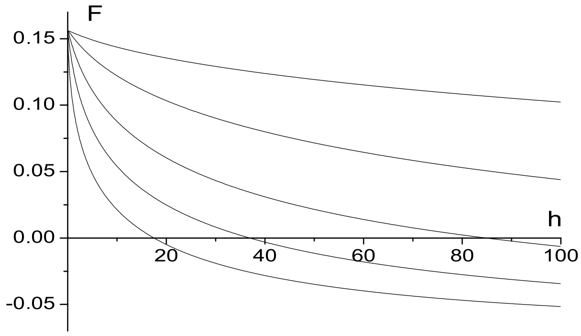

Figure 3. The dependence of the amplification coefficient F on the parameter h for fixed values of the parameters $A_0 = 10$, $Z = 0.3$ and different values of parameters $g = 0.1, 0.3, 0.8, 2.0, 10.0$ (from top to bottom).

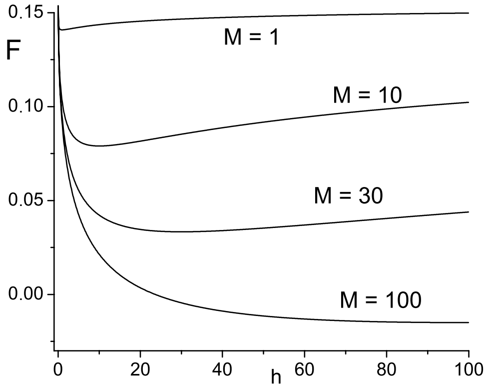

Figure 4. The dependence of the amplification coefficient F on the parameter h for fixed values of the parameter $M = gh = R\alpha/\omega_0$.

the layer nearby the surface where the carriers were created by the short laser pulse is α^{-1}. The layer spreads due to the diffusion by the distance of the order of $\sqrt{Y/\omega_0}$ during the time interval ω_0^{-1}. If this distance exceeds α^{-1}, then the deformation of the initial profile of the concentration $n(x)$ becomes too fast, and the effective 'reflection coefficient' $|r|$ becomes too small. Of course, the concrete critical value $h \sim 1$ cannot be found from these qualitative considerations, and it arises as a result of numerical calculations of the integrals in (13). Using some special procedures (e.g., etching the surface) one can reduce the surface recombination velocity to the values of the order of 10^2–10^3 cm s^{-1} [58]. In such a case one has $g < 1$ for $\alpha > 10^2$ cm^{-1}, so that the influence of g and h becomes insignificant. This special case ($g = 0$) was studied in [45].

One can also diminish the parameter g by means of increasing the diffusion coefficient (or the mobility). But in this case the parameter h *increases* in the same proportion. For this reason, we show in figure 4 the dependence $F(h)$ for the fixed values of A_0, Z and the parameter $M = gh = R\alpha/\omega_0$. All the curves start at the same value $F \approx 0.15$ for $h \to 0$, but the function $F(h)$ rapidly decreases with increase of h (i.e., mobility in the case of fixed values of the surface recombination velocity and the absorption coefficient) if $M \gg 1$. Moreover, this function becomes negative for $h > 20$ and $M = 100$. This example demonstrates again that one should avoid big values of α and R, trying to maintain $M < 1$. For instance, the preferable values of α are less than 10^3 cm^{-1} for 'bad' surfaces with $R \sim 10^7$ cm s^{-1}. Or one should keep $R < 10^4$ cm s^{-1} if $\alpha \sim 10^6$ cm^{-1}.

5. Discussion

For the mobility of carriers $b \sim 1$ m^2 V^{-1} s^{-1} and the dimensions of the rectangular cavity reported in [37] (the length $L = 11$ cm, the biggest transverse dimension $B = 7$ cm and the smallest dimension 2 cm, so that $S = 14$ cm^2) the value $A_0 = 10$ corresponds to the energy of individual pulse ∼2 mJ, if $\Delta = 0.01$ (i.e., the slab thickness $D = 0.6$ mm) and $E_s = 1.4$ eV (as for GaAs). This value of A_0 is close to $A_{0*} = 11.3$, which gives the minimum (in the case $g = 0$) of the function $A_0/F(A_0, Z)$ proportional to the *total energy* of $n = \mathrm{const}/F$ pulses necessary to create a given number of photons for the fixed geometry (see [45]; the corresponding values of other parameters are $Z_* = 0.29$ and $F_* = 0.18$). On the other hand, $A_0 = 10$ corresponds to the effective surface concentration (24) immediately after the laser pulse $\mathcal{K}_0 \approx 5 \times 10^{14}$ cm^{-2} (for the same mobility and the slab thickness). The geometry chosen gives the values $\eta = 0.55$ and $\eta^3 = 0.16$. Then one can easily calculate that, say, $\mathcal{N} = 10^3$ photons can be generated after $n \approx 12\,000$ pulses in the case of ideal surface. Looking at figure 2, we see that the surface recombination can reduce the maximal value of F for almost two times in the case of $g = 10$ and $h = 1$ (and the same value $A_0 = 10$), thus increasing by almost two times the number of pulses and total energy (which would be about 40 J). This example shows the importance of using very good surfaces and materials with not too big values of the absorption coefficient $\alpha < 10^5$ cm^{-1}. Besides, the dimensionless parameter Z must be of the order of 0.3, which corresponds to the recombination time $T_r = 20$ ps for $f_0 = 2.5$ GHz. As was reported in [60], it is possible to reduce the recombination time in GaAs up to 3 ps, using the implantation of ions Au^+. Even more short recombination times, less than 1 ps, were reported (for other materials) in [61]. Consequently, the value $T_r^{opt} \approx 20$ ps is quite realistic from the point of view of the available technology.

However, the total energy of all pulses is obviously too high (remember that this energy will be released inside the semiconductor after the recombination and heat it, so it must be quickly removed). A possible way to diminish the total energy and the number of laser pulses is to search for semiconductors with very high mobility of carriers (because the parameter A_0 is proportional to the product $W|b|$). The maximal mobility achieved in semiconductor heterostructures based on GaAs exceeds the value 10^3 m^2 V^{-1} s^{-1} for temperatures of the order of 1–4 K [62]. It was obtained for the two-dimensional electron gas in small samples (with the area ∼5×5 mm^2 and thickness 10^{-6}–10^{-5} cm) with the aid of illumination from a red light-emitting diode. Note that the absorption length (i.e., the initial width of a highly conducting layer nearby the semiconductor surface) has the same order of magnitude for the absorption coefficient $\alpha \sim 10^6$–10^5 cm^{-1}, and the use of TE polarization in the experiment on NSCE means that one needs high conductivity, namely, in the direction parallel to the surface. It is not clear, however, whether the reported big values of conductivity are compatible with the necessary small recombination time, which can be achieved by means of strong doping. But if such materials could be found (with a big surface area and small surface recombination velocity), this would resolve the problem, because the necessary total energy of laser pulses would be 100–1000 times smaller, i.e., about 0.1 J or less.

Besides, one can change the geometry. The simplest possibility is to increase the thickness D of the slab (remember that $\Delta = 2D/\lambda$) or to put the semiconductor plate on a relatively thick dielectric substrate with high thermal conductivity. Increasing the parameter Δ from 0.01 to 0.1 could result in diminishing the necessary number of pulses and the total energy by

10 times. A further increase of the thickness hardly could help, because the approximation used seems to be reasonable only under the condition $\nu^2 n \ll 1$ (as it happens in the standard parametric resonance case, when the function $\omega(t)$ performs small *harmonical* oscillations). For $\Delta > 0.1$, we expect that the influence of the neglected nonlinear terms in the equations of motion and the interaction with other modes in the cavity could be important. (For the numerical verification of the accuracy of different analytical approximations, see [43].) Some gain can be also achieved by means of the optimization of the cavity shape, which would result in bigger values of the geometrical factor η (remember that it enters the formulae as η^3) [45, 46]. The generation of photons from the initial thermal state (instead of the vacuum) could also reduce the total energy of laser pulses [39, 46].

We have demonstrated that the cavity QED in the presence of boundaries with time-dependent parameters is a very interesting area of research, both for theoreticians and experimentalists. We hope that the combined efforts of specialists working in different fields (quantum mechanics of open systems, laser physics, semiconductor physics, electrodynamics of cavities with non-trivial geometry and material properties of walls, etc) will resolve the problems pointed out above (and many other, not concerned here, such as the non-uniform illumination of the surface, possible detection schemes, etc), and the photon generation from vacuum in cavities due to the non-stationary Casimir effect will be observed in the not very remote future.

Acknowledgments

VVD acknowledges the partial support of the Brazilian agency CNPq. AVD acknowledges the full support of the Brazilian agency FAPESP, project 04/13705-3.

Appendix. Solving equation (25) for the spacetime concentration of photo-carriers

Supposing that all functions depend on the only spatial variable x and introducing the notation

$$n(x,t) = \tilde{n}(x,t)\exp(-\beta_1 t), \qquad f(x,t) = (\alpha\zeta/E_s)I(t)\exp(\beta_1 t - \alpha x) \tag{A.1}$$

we rewrite equation (25) (for $Y = \text{const}$) as

$$\partial\tilde{n}/\partial t = Y\partial^2\tilde{n}/\partial x^2 + f(x,t). \tag{A.2}$$

First, we look for the complete set of solutions to the *homogeneous counterpart* of equation (A.2). They can be represented as $\tilde{n}_k(x,t) = \exp(-Yk^2 t)\psi_k(x)$, where the functions

$$\psi_k(x) = \frac{1}{\sqrt{2\pi}}\left[e^{ikx} + \frac{kY + iR}{kY - iR}e^{-ikx}\right] \tag{A.3}$$

satisfy the equation $\partial^2\psi_k/\partial x^2 = -k^2\psi_k$ in the half-space $x > 0$, the boundary condition (26) and the orthogonality condition

$$\int_0^\infty \psi_k^*(x)\psi_j(x)\,dx = \delta(k - j). \tag{A.4}$$

The continuous index k can assume any *positive* value.

Using the completeness relation

$$\int_0^\infty \psi_k^*(x)\psi_k(y)\,dk = \delta(x - y), \tag{A.5}$$

we look for the solution of the full inhomogeneous equation (A.2) in the form

$$\tilde{n}(x,t) = \int_0^\infty c_k(t)\psi_k(x)\,dk. \tag{A.6}$$

In such a case the function $c_k(t)$ must obey the equation

$$\dot{c}_k = -Yk^2 c_k + b_k(t), \qquad b_k(t) = \int_0^\infty \psi_k^*(x) f(x,t)\,\mathrm{d}x \tag{A.7}$$

whose solution satisfying the initial condition $c_k(-\infty) = 0$ reads (we suppose that the initial concentration of carriers was equal to zero; in the generic case $n(x,t)$ means the deviation of the concentration from the initial thermal equilibrium value)

$$c_k(t) = \int_{-\infty}^{t} \mathrm{d}\tau \int_0^\infty \psi_k^*(y) f(y,\tau) \exp[-Yk^2(t-\tau)]\,\mathrm{d}y. \tag{A.8}$$

Combining (A.1), (A.3), (A.6), (A.8) and introducing the dimensionless integration variable $\kappa = k/\alpha$, we arrive after simple algebra to the solution of equation (25) in the form given by equations (27) and (28). The integral in (28) can be easily expressed in terms of the complementary error function in the limit cases of $g = 0$ and $g = \infty$:

$$\Psi_0(x,t') = \frac{1}{2}\,\mathrm{e}^{\alpha^2 Yt'}\left[\mathrm{e}^{-\alpha x}\,\mathrm{Erfc}\left(\frac{2\alpha^2 Yt' - \alpha x}{2\sqrt{\alpha^2 Yt'}}\right) + \mathrm{e}^{\alpha x}\,\mathrm{Erfc}\left(\frac{2\alpha^2 Yt' + \alpha x}{2\sqrt{\alpha^2 Yt'}}\right)\right] \tag{A.9}$$

$$\Psi_\infty(x,t') = \frac{1}{2}\,\mathrm{e}^{\alpha^2 Yt'}\left[\mathrm{e}^{-\alpha x}\,\mathrm{Erfc}\left(\frac{2\alpha^2 Yt' - \alpha x}{2\sqrt{\alpha^2 Yt'}}\right) - \mathrm{e}^{\alpha x}\,\mathrm{Erfc}\left(\frac{2\alpha^2 Yt' + \alpha x}{2\sqrt{\alpha^2 Yt'}}\right)\right] \tag{A.10}$$

(formally, the difference is only in the sign of the second term, but it results in quite different distributions of the concentration).

The integrals (27) and (28) can easily be calculated for the δ-pulse $I(t) = (W/S)\delta(t)$ immediately after the pulse (when one can neglect the term $-\kappa^2\alpha^2 Yt$ in the argument of the exponential function, so that one can extend the integration over κ to the whole real axis, close the integration contour in the upper semi-plane and apply the theory of residues). In this case, we have

$$n(x,0+) = \frac{W\zeta\alpha}{SE_\mathrm{s}}\left[\mathrm{e}^{-\alpha x} - \frac{g}{1+g}\,\mathrm{e}^{-g\alpha x}\right], \qquad n(0,0+) = \frac{W\zeta\alpha}{SE_\mathrm{s}(1+g)}. \tag{A.11}$$

This formula clearly shows that the surface recombination significantly diminishes the initial concentration of carriers nearby the surface if $g > 1$ (although it does not change the initial profile at the distances satisfying the relation $g\alpha x \gg 1$).

References

[1] Bialynicka-Birula Z and Bialynicki-Birula I 1987 Space-time description of squeezing *J. Opt. Soc. Am.* B **4** 1621–6

[2] Lobashov A A and Mostepanenko V M 1991 Quantum effects associated with parametric generation of light and the theory of squeezed states *Teor. Mat. Fiz.* **88** 340–57

Lobashov A A and Mostepanenko V M 1991 Quantum effects associated with parametric generation of light and the theory of squeezed states *Theor. Math. Phys.* **88** 913–25 (Engl. Transl.)

[3] Dodonov V V, George T F, Man'ko O V, Um C-I and Yeon K-H 1992 Exact solutions for a mode of the electromagnetic field in resonator with time-dependent characteristics of the internal medium *J. Sov. Laser Res.* **13** 219–30

[4] Dodonov V V, Klimov A B and Nikonov D E 1993 Quantum phenomena in nonstationary media *Phys. Rev.* A **47** 4422–9

[5] Dodonov V V and Man'ko V I 1994 Correlated and squeezed coherent states of time-dependent quantum systems *Modern Nonlinear Optics (Advances in Chemical Physics Series* vol 85 part 3*)* ed M Evans and S Kielich (New York: Wiley) pp 499–530

[6] Artoni M, Bulatov A and Birman J 1996 Zero-point noise in a nonstationary dielectric cavity *Phys. Rev.* A **53** 1031–5

Artoni M, Bulatov A and Seery B D 1998 Nonclassical phase of the electromagnetic field in a nonstationary dielectric *Phys. Rev.* A **58** 3345–8

[7] Yablonovitch E 1989 Accelerating reference frame for electromagnetic waves in a rapidly growing plasma: Unruh–Davies–Fulling–De Witt radiation and the nonadiabatic Casimir effect *Phys. Rev. Lett.* **62** 1742–5

[8] Hizhnyakov V V 1992 Quantum emission of a medium with a time-dependent refractive index *Quantum Opt.* **4** 277–80

[9] Schwinger J 1993 Casimir light: a glimpse *Proc. Natl Acad. Sci. USA* **90** 958–9

[10] Johnston H and Sarkar S 1995 Moving mirrors and time-varying dielectrics *Phys. Rev.* A **51** 4109–15

[11] Okushima T and Shimizu A 1995 Photon emission from a false vacuum of semiconductors *Japan. J. Appl. Phys.* **34** 4508–10

[12] Saito H and Hyuga H 1996 Photon emission from a time-varying dielectric medium in a cavity *J. Phys. Soc. Japan* **65** 1139–42

Saito H and Hyuga H 1996 The dynamical Casimir effect for an oscillating dielectric model *J. Phys. Soc. Japan* **65** 3513–23

[13] Shimizu A, Okushima T and Koshino K 1997 Quantum optics in dispersive and absorptive media *Mater. Sci. Eng.* B **48** 66–70

[14] Eberlein C 1999 Quantum radiation from density variations in dielectrics *J. Phys. A: Math. Gen.* **32** 2583–8

[15] Barton G and Eberlein C 1993 On quantum radiation from a moving body with finite refractive index *Ann. Phys.* **227** 222–74

[16] Barton G and Calogeracos A 1995 On the quantum electrodynamics of a dispersive mirror: I. Mass shifts, radiation, and radiative reaction *Ann. Phys.* **238** 227–67

[17] Salamone G M and Barton G 1995 Quantum radiative reaction on a dispersive mirror in one dimension *Phys. Rev.* A **51** 3506–12

[18] Barton G 1996 The quantum radiation from mirrors moving sideways *Ann. Phys.* **245** 361–88

[19] Barton G and North C A 1996 Peculiarities of quantum radiation in three dimensions from moving mirrors with high refractive index *Ann. Phys.* **252** 72–114

[20] Gütig R and Eberlein C 1998 Quantum radiation from moving dielectrics in two, three, and more spatial dimensions *J. Phys. A: Math. Gen.* **31** 6819–38

[21] Pendry J B 1998 Can sheared surfaces emit light? *J. Mod. Opt.* **45** 2389–408

[22] Guerreiro A, Mendonça J T and Martins A M 2005 New mechanism of vacuum radiation from non-accelerated moving boundaries *J. Opt. B: Quantum Semiclass. Opt.* **7** S69–76

[23] Mendonça J T, Guerreiro A and Martins A M 2000 Quantum theory of time refraction *Phys. Rev.* A **62** 033805

Mendonça J T and Guerreiro A 2005 Time refraction and the quantum properties of vacuum *Phys. Rev.* A **72** 063805

[24] Hussain N, Imoto N and Loudon R 1992 Quantum theory of dynamic interference experiments *Phys. Rev.* A **45** 1987–96

[25] Imoto N 1997 Quantum effects of spatial/temporal modulation of the optical field *Mater. Sci. Eng.* B **48** 34–8

[26] Mendonça J T, Martins A M and Guerreiro A 2003 Temporal beam splitter and temporal interference *Phys. Rev.* A **68** 043801

[27] Braunstein S L 2005 A quantum optical shutter *J. Opt. B: Quantum Semiclass. Opt.* **7** S28–31

[28] Cirone M, Rzążewski K and Mostowski J 1997 Photon generation by time-dependent dielectric: a soluble model *Phys. Rev.* A **55** 62–6

[29] Cirone M and Rzążewski K 1999 Electromagnetic radiation in a cavity with a time-dependent mirror *Phys. Rev.* A **60** 886–92

[30] Crocce M, Dalvit D A R, Lombardo F C and Mazzitelli F D 2004 Model for resonant photon creation in a cavity with time-dependent conductivity *Phys. Rev.* A **70** 033811

[31] Uhlmann M, Plunien G, Schützhold R and Soff G 2004 Resonant cavity photon creation via the dynamical Casimir effect *Phys. Rev. Lett.* **93** 193601

[32] Man'ko V I 1992 Time-dependent invariants and nonclassical light *Quantum Measurements in Optics (NATO ASI Series B: Physics* vol 282*)* ed P Tombesi and D F Walls (New York: Plenum) pp 239–46

[33] Grishchuk L P, Haus H A and Bergman K 1992 Generation of squeezed radiation from vacuum in the cosmos and the laboratory *Phys. Rev.* D **46** 1440–9

[34] Lozovik Y E, Tsvetus V G and Vinogradov E A 1995 Parametric excitation of vacuum by use of femtosecond laser pulses *Phys. Scr.* **52** 184–90

[35] Braggio C, Bressi G, Carugno G, Lombardi A, Palmieri A, Ruoso G and Zanello D 2004 Semiconductor microwave mirror for a measurement of the dynamical Casimir effect *Rev. Sci. Instrum.* **75** 4967–70

[36] Macchi A, Cornolti F, Pegoraro F, Liseikina T V, Ruhl H and Vshivkov V A 2001 Surface oscillations in overdense plasmas irradiated by ultrashort laser pulses *Phys. Rev. Lett.* **87** 205004

[37] Braggio C, Bressi G, Carugno G, Del Noce C, Galeazzi G, Lombardi A, Palmieri A, Ruoso G and Zanello D 2005 A novel experimental approach for the detection of the dynamic Casimir effect *Europhys. Lett.* **70** 754–60

[38] Dodonov V V 1995 Photon creation and excitation of a detector in a cavity with a resonantly vibrating wall *Phys. Lett.* A **207** 126–32
Dodonov V V and Klimov A B 1996 Generation and detection of photons in a cavity with a resonantly oscillating boundary *Phys. Rev.* A **53** 2664–82

[39] Plunien G, Schützhold R and Soff G 2000 Dynamical Casimir effect at finite temperature *Phys. Rev. Lett.* **84** 1882–5

[40] Crocce M, Dalvit D A R and Mazzitelli F D 2001 Resonant photon creation in a three-dimensional oscillating cavity *Phys. Rev.* A **64** 013808

[41] Crocce M, Dalvit D A R and Mazzitelli F D 2002 Quantum electromagnetic field in a three-dimensional oscillating cavity *Phys. Rev.* A **66** 033811

[42] Crocce M, Dalvit D A R, Lombardo F C and Mazzitelli F D 2005 Hertz potentials approach to the dynamical Casimir effect in cylindrical cavities of arbitrary section *J. Opt. B: Quantum Semiclass. Opt.* **7** S32–9

[43] Ruser M 2005 Vibrating cavities: a numerical approach *J. Opt. B: Quantum Semiclass. Opt.* **7** S100–15
Ruser M 2005 A numerical approach to the dynamical Casimir effect *Preprint* quant-ph/0509030

[44] Dodonov V V 2005 Time-dependent quantum damped oscillator with 'minimal noise': application to the nonstationary Casimir effect in non-ideal cavities *J. Opt. B: Quantum Semiclass. Opt.* **7** S445–51

[45] Dodonov V V and Dodonov A V 2005 Quantum harmonic oscillator and nonstationary Casimir effect *J. Russ. Laser Res.* **26** 445–83

[46] Dodonov V V and Dodonov A V 2006 Nonstationary Casimir effect in a cavity with periodical time-dependent conductivity of a semiconductor mirror *J. Phys. A: Math. Gen.* **39** at press

[47] Man'ko V I 1991 The Casimir effect and quantum vacuum generator *J. Sov. Laser Res.* **12** 383–5
Dodonov V V, Man'ko O V and Man'ko V I 1993 Correlated states of string and gravitational waveguide *Squeezed and Correlated States of Quantum Systems (Proc. Lebedev Physics Institute* vol 205*)* ed M A Markov (Commack, NY: Nova Science) pp 163–229

[48] Sassaroli E, Srivastava Y N and Widom A 1994 Photon production by the dynamical Casimir effect *Phys. Rev.* A **50** 1027–34

[49] Schützhold R, Plunien G and Soff G 1998 Trembling cavities in the canonical approach *Phys. Rev.* A **57** 2311–8

[50] Senitzky I R 1960 Dissipation in quantum mechanics. The harmonic oscillator *Phys. Rev.* **119** 670–9
Schwinger J 1961 Brownian motion of a quantum oscillator *J. Math. Phys.* **2** 407–32
Haus H A and Mullen J A 1962 Quantum noise in linear amplifiers *Phys. Rev.* **128** 2407–13
Lax M 1966 Quantum noise: IV. Quantum theory of noise sources *Phys. Rev.* **145** 110–29

[51] Gruner T and Welsch D-G 1996 Green-function approach to the radiation-field quantization for homogeneous and inhomogeneous Kramers–Kronig dielectrics *Phys. Rev.* A **53** 1818–29
Tip A, Knöll L, Scheel S and Welsch D-G 2001 Equivalence of the Langevin and auxiliary-field quantization methods for absorbing dielectrics *Phys. Rev.* A **63** 043806
Cheng Y J and Siegman A E 2003 Generalized radiation-field quantization method and the Petermann excess-noise factor *Phys. Rev.* A **68** 043808

[52] Ginzburg V L 1967 *Propagation of Electromagnetic Waves in Plasmas* (Moscow: Nauka) pp 292–4

[53] Dodonov A V and Dodonov V V 2005 Resonance generation of photons from vacuum in cavities due to strong periodical changes of conductivity in a thin semiconductor boundary layer *J. Opt. B: Quantum Semiclass. Opt.* **7** S47–58

[54] Laurich B, Hillmer H and Forchel A 1987 Optical time-of-flight investigation of the exciton transport in silicon *J. Appl. Phys.* **61** 1480–5

[55] Vogel T, Dodel G, Holzhauer E, Salzmann H and Theurer A 1992 High-speed switching of far-infrared radiation by photoionization in a semiconductor *Appl. Opt.* **31** 329–37

[56] Wilson T E 1999 Modeling the high-speed sweetching of far-infrared radiation by photoionization in a semiconductor *Phys. Rev.* B **59** 12996–13002

[57] Inui N 2004 Numerical study of enhancement of the Casimir force between silicon membranes by irradiation with UV laser *J. Phys. Soc. Japan* **73** 332–9

[58] Aspnes D E 1983 Recombination at semiconductor surfaces and interfaces *Surf. Sci.* **132** 406–21

[59] Oberhettinger F and Badii L 1973 *Tables of Laplace Transforms* (Berlin: Springer)

[60] Mangeney J, Stelmakh N, Aniel F, Boucaud P and Lourtioz J-M 2002 Temperature dependence of the absorption saturation relaxation time in light- and heavy-ion-irradiated bulk GaAs *Appl. Phys. Lett.* **80** 4711–3

[61] Sukhotin M, Brown E R, Driscoll D, Hanson M and Gossard A C 2003 Picosecond photocarrier-lifetime in ErAs:InGaAs at 1.55 μm *Appl. Phys. Lett.* **83** 3921–3

[62] Zudov M A, Du R R, Pfeiffer L N and West K W 2003 Evidence for a new dissipationless effect in 2D electronic transport *Phys. Rev. Lett.* **90** 046807
Mani R G, Smet J H, von Klitzing K, Narayanamurti V, Johnson W B and Umansky V 2004 Radiation-induced oscillatory magnetoresistance as a sensitive probe of the zero-field spin-splitting in high-mobility GaAs/Al_xGa_{1-x}As devices *Phys. Rev.* B **69** 193304
Studenikin S A, Potemski M, Sachrajda A, Hilke M, Pfeiffer L N and West K W 2005 Microwave-induced resistance oscillations on a high-mobility two-dimensional electron gas: exact waveform, absorption/reflection and temperature damping *Phys. Rev.* B **71** 245313