Teleporting a state inside a single bimodal high-$Q$ cavity

Geisa Pires, A. T. Avelar, B. Baseia, and N. G. de Almeida

1Instituto de Física, Universidade Federal de Goiás, 74.001-970, Goiânia (GO) Brazil
2Instituto de Física, Universidade de Brasília, 70.919-970, Brasília, (DF) Brazil
3Núcleo de Pesquisas em Física, Universidade Católica de Goiás, 74.605-220, Goiânia (GO), Brazil

(Rceived 19 October 2004; published 1 June 2005)

We discuss a simplified scheme to teleport a state from one mode to another of the same bimodal cavity, with these two modes having distinct frequencies and orthogonal polarizations. The scheme employs two two-level (Rydberg) atoms plus classical fields (Ramsey zones) and selective atomic state detectors. The result has potential use for the manipulation of quantum information processing.

DOI: 10.1103/PhysRevA.71.060301 PACS number(s): 03.67.–a, 03.65.Ta, 42.50.Dv

I. INTRODUCTION

Since the proposal by Bennett et al. [1] the teleportation phenomenon has received great attention and a number of protocols have been suggested for its implementation in different contexts, as for trapped ions [2], running waves [3,4], and cavity QED [5]. Experimentally, teleportation has been demonstrated for discrete variables [6–8] and a single-mode field of continuous variables [9]. The main challenge consists of the so-called Bell state measurement, performed on the Bell operator basis for the particle whose state is to be teleported, plus its partner composing the quantum channel [1].

In the realm of cavity QED, schemes for teleporting twoparticle [10], multiparticle entangled atomic states [11], and also entangled field states inside high-$Q$ cavities [12] have been proposed. In Ref. [12] the authors studied a scheme to teleport an entangled state composed by a two-qubit field state (readily generalized for $N$ qubit) from a pair of high-$Q$ cavities to another pair of such cavities. Recently, more economical schemes were studied, opening the way to test teleportation in the QED scenario: one of them [13] teleporting a two-mode entangled state from one cavity to another with a success probability of 100%; another one [14] teleporting the state of a high-$Q$ cavity to another with a success probability of 25%. The scheme in Ref. [14] differs from the original one [1], since it is not based on Bell measurements; thus the collapse due to measurement on states of two-level atoms projects the state to be teleported from one cavity to another only in an approximate way, the fidelity of the teleportation being not unity.

Although the foregoing schemes using high-$Q$ cavities represent advances by simplifying the required procedures to teleport states of cavity modes, all experiments implemented until now involve only a single cavity, due to both decoherence and complexity of experimental challenges. However, cavity is an important scenario for testing fundamentals of quantum mechanics [15] as well as for demonstrating quantum information processing [16]; hence experiments involving teleportation—the cornerstone of universal quantum computation [17,18]—are expected to be soon reported in the context of high-$Q$ cavities.

Here we propose an oversimplified scheme to teleport a state from one mode to another of a single bimodal high-$Q$ cavity. Teleporting a superposed state here has the same meaning as teleporting an arbitrary state, and the two modes constitute the two systems (Alice and Bob) of the traditional scheme. The two modes have different frequencies and polarizations. The cavity is the same, but its different modes occupy distinct regions inside it. Originally [1], the effect has been understood as occurring at very large distances; the reason being that usually the schemes employ particles having interactions that vanish when they are far away. Such a requirement is not necessary for photons, which are noninteracting particles, no matter the distance between them. Later on, the effect was so recognized at arbitrary distances, e.g., at 1-m distance, as in an earlier experiment [6]. After that [17,18], the effect has been applied in quantum computation, which would extend it even to small distances, the effect being identified by its mechanisms (preparation of a state; preparation of a quantum channel between two systems; plus a joint measurement). “Large distances” will be common in quantum communication, whereas “small distances” stand for quantum computation. In the latter case, reducing the distance attenuates the source of errors due to decoherence effects caused by action of environments. Actually, in the present work the two modes are distinguished by their frequencies and their orthogonal polarizations [16]; hence they occupy distinct regions in the cavity and we are talking about teleportation from a region occupied by one mode to another (neighboring) region occupied by another mode of the same cavity. The crucial ingredient characterizing the effect is the transfer of information between noninteracting systems (the two neighboring modes), at the expense of a quantum channel. In the words of Brassard et al. [18]: “…indeed its primary purpose (of teleportation) is increased computational speed—and within a small region of space; …it may be that short-distance quantum teleportation will play a role in transporting quantum information inside quantum computers. Thus we see that the fates of quantum computing and quantum teleportation are inseparably entangled!”

The simplicity of our scheme makes it attractive experimentally, and it is feasible with present QED technology. Our scheme is based on a recent experimental report concerned with the controlled entanglement of two field modes...
in a high-$Q$ cavity [16]. Similar to Ref. [16], our proposed scheme uses circular Rydberg atoms and two modes of a cavity, plus Ramsey zones and selective atomic state detectors. By following the original protocol [1], the success probability is 50%. As assumed in [12–14], the whole losses due to atomic spontaneous emission and dissipation in the cavities are neglected. In fact, since the decoherence time is of the same order of the lifetimes for the qubits in a high-$Q$ cavity and the (spontaneous) atomic decay, the present scheme should be realized during the $10^{-2}$ s, a typical time for both decoherence and damping of atomic and cavity qubits [19].

II. CONTROLLED INTERACTIONS

To perform teleportation of the zero- and one-photon entangled state between two modes of a high-$Q$ cavity, we will need the following operators:

$$H_{on} = \hbar g (\sigma^+ a + \sigma^- a^*),$$

(1)

$$H_{off} = \left( \frac{\hbar g^2}{\delta} \right) a^* a |e\rangle \langle e|,$$

(2)

$$R = (I - i \sigma_1)/\sqrt{2}.$$  \hspace{1cm} (3)

Equation (1) is the usual Jaynes-Cummings model [21] and describes a resonant interaction of mode $a$ represented by creation and annihilation operators $(a,a^*)$ with a two-level atom represented by the Pauli operators $\sigma^+$’s; $g$ is the atom-field coupling constant. Equation (2) stands for dispersive atom-field interaction [21] and can be implemented via a Stark shift; $\delta=(\omega-a_0)$ is the detuning between the field frequency $\omega$ and the atomic frequency $a_0$. Equation (3) represents the action of the Ramsey zone [19]. Using the operators defined in Eqs. (1)–(3), it becomes easy to verify the following evolutions:

$$R \left[ \begin{array}{c} |g\rangle \\ |e\rangle \end{array} \right] = \frac{1}{\sqrt{2}} \left[ \begin{array}{c} |g\rangle - |e\rangle \\ |g\rangle + |e\rangle \end{array} \right] = |\pm\rangle,$$

(4)

$$U_{on}(\alpha,\beta) \left[ \begin{array}{c} |a\rangle_0 |0\rangle_\beta \\ |a\rangle_0 |1\rangle_\beta \\ |a^\dagger\rangle_0 |0\rangle_\beta \\ |a^\dagger\rangle_0 |1\rangle_\beta \end{array} \right],$$

(5)

$$U_{on}(\alpha,\beta) \left[ \begin{array}{c} |+\rangle_1 |0\rangle_\beta \\ |+\rangle_1 |1\rangle_\beta \\ |\rangle_1 |0\rangle_\beta \\ |\rangle_1 |1\rangle_\beta \end{array} \right] = U_{off}(\alpha,\beta) \left[ \begin{array}{c} |\pm\rangle_0 |0\rangle_\beta \\ |\pm\rangle_0 |1\rangle_\beta \\ |\rangle_0 |0\rangle_\beta \\ |\rangle_0 |1\rangle_\beta \end{array} \right].$$

(6)

We note that the evolutions $U_{on}(\alpha,\beta)$ and $U_{off}(\alpha,\beta)$ are obtained by adjusting the interaction times as $g t=\pi/2$ and $g^2 t/\delta=\pi$ from $U_{on}(\alpha,\beta)=\exp[-(i/\hbar)H_{on}t]$ and $U_{off}(\alpha,\beta)=\exp[-(i/\hbar)H_{off}t]$, respectively. The subindex $\alpha$ stands for atoms—$\alpha$ in ground and excited states $(g,e)$, and $\beta$ represents the cavity modes $(a,b)$. Here we are assuming the operation $U(1,a)$ leaving mode $b$ unaffected. Indeed, this is a crucial point, since relative phases will appear necessarily from atom-field interactions. This phase was proved to be irrelevant in Ref. [13] by the following reasoning: as is readily seen from the Hamiltonian model for a two-level atom interacting resonantly with, say, mode $b$ and off-resonantly with mode $a$, the effective Hamiltonian leads to an energy shift corresponding to mode $a$. This shifting is inversely proportional to the detuning $\delta=(a_\alpha-a_\beta)$ between the field frequency $a_\alpha$ of mode $a$ and the atomic frequency $a_\beta$ [see Eq. (2)]. Thus, if $\delta$ is large enough compared with the atom-field coupling parameter $g$ we can neglect the effect of the off-resonant interaction. To estimate the additional phases gained due to dispersive interaction between one of the two modes $(a_\alpha,a_\beta)$ of the cavity and the atomic frequency of the two-level atom, let us consider the atom-field interactions such that $a_\alpha=a_\beta>\omega_a$ and $g t=\pi/2$. Thus, the mode $a$ will be shifted by the phase $\phi=g^2 t((a_\alpha-a_\beta))=\pi g/2(a_\beta-a_\alpha)$. Next, assume a Stark shift on mode $b$ leading to $a_\beta=a_\alpha+\delta$ in such a way that $g^2 t/\delta=\pi$. In this case, mode $a$ is shifted by the phase $\phi'=(g^2 t/((a_\alpha-a_\beta))\pm\delta$ and taking the experimental values [19] $a_\alpha-a_\beta=10^{10}$ Hz, $g=10^3$ Hz, $\delta=10$ g, we find that $\phi=\phi'=10^{-5}$ rad, which can be safely neglected.

III. ORIGINAL PROTOCOL TELEPORTATION

A. Preparing the state to be teleported in mode A

Here we take advantage of the experiment in Ref. [16]: a two-level atom $A_1$ crosses a Ramsey zone $R(\theta)$, $\theta$ being an arbitrary rotation, and interacts resonantly first with mode $M_a$; after that atom $A_1$ is tuned to the Stark effect, which makes the atom resonant to a classical field frequency $\Omega$, causing the rotation $\pi/2$ of the atomic state [20], as given in Eq. (4); next, atom $A_1$ interacts resonantly with mode $M_b$ letting the atom in ground state $|g\rangle$. These steps are shown below,

$$R(\theta)|e\rangle_1 |0\rangle_0 |0\rangle_b \rightarrow (C_1 |g\rangle_1 + C_2 |e\rangle_1) |0\rangle_0 |0\rangle_b,$$

(7)

$$U_{on}(1,a)(C_1 |g\rangle_1 + C_2 |e\rangle_1) |0\rangle_0 |0\rangle_b \rightarrow |\psi_\alpha\rangle_0 |a\rangle_0 |g\rangle_1,$$

(8)

$$|\psi_\alpha\rangle_0 |g\rangle_1 R(\theta)|e\rangle_1 \rightarrow |\psi_\alpha\rangle_0 \frac{1}{\sqrt{2}}(|g\rangle_a - |e\rangle_a) |0\rangle_b,$$

(9)

$$|\psi_\alpha|U_{on}(1,b)(|g\rangle_1 - |e\rangle_1) |0\rangle_b \rightarrow |\psi_\alpha|_0 |0\rangle_b - |1\rangle_b |g\rangle_1,$$

(10)

where $|\psi_\alpha\rangle=(C_1 |0\rangle_a + C_2 |1\rangle_a)$. In Fig. 1 the temporal region $[0,\tau_1]$ stands for the steps of the first atom $A_1$, as mentioned above.

Note that without the strategy of producing a rotation in the atomic state inside the cavity, another atom would be needed to produce the state $(|0\rangle_a - |1\rangle_a)/\sqrt{2}$ in mode $b$. Making this atom in state $|e\rangle$ to cross the Ramsey zone and cavity, the system evolves according to Eqs. (4) and (5), namely: $|\psi_\alpha\rangle=(1/\sqrt{2})(|0\rangle_b - |1\rangle_b)|g\rangle$. This superposition in mode $b$ is necessary to accomplish the nonlocal channel as we will de-


060301-2
scribe in the next step. Nevertheless, a rotation $\pi/2$ applied to the atom during its passage throughout the cavity is in principle possible [20], since the detuning can be set to a large value, freezing the atom-field evolution [16] while the atom is rotated by the classical field.

**B. Preparing the nonlocal channel**

Next, atom $A_1$ will play the following roles: (i) it composes the nonlocal channel with mode $b$; (ii) it establishes Bell states with mode $a$; (iii) it interacts off resonantly with mode $a$ and evolves according to Eq. (6), as a partial measurement allowing one to distinguish, among the Bell states, the states $|\Psi^\pm\rangle$ from states $|\Phi^\pm\rangle$. To prepare the nonlocal channel, the atom $A_1$ is tuned to the classical field, which leads it to the superposed state $|(e_1 + |g_1)/\sqrt{2}\rangle$. Then another controlled Stark effect is applied in order to let atom $A_1$ interact off resonantly with mode $b$, according to Eq. (6). The result is the nonlocal channel, 

$$|\chi\rangle_{NL} = (|+\rangle_1|0\rangle_b - |\ast\rangle_b|1\rangle_1)/\sqrt{2}. \quad (11)$$

The state $|\Psi\rangle_T = |\chi\rangle_{NL}|\psi\rangle_a$ describing the whole system reads 

$$|\Psi\rangle_T = \frac{1}{\sqrt{2}}(C_1|0\rangle_a + \lambda_1|0\rangle_b - C_1|0\rangle_a - \lambda_1|1\rangle_b + C_2|1\rangle_a + \lambda_1|0\rangle_b - C_2|1\rangle_a - \lambda_1|1\rangle_b). \quad (12)$$

If we define the states of the Bell basis composed by atom $A_1$ and mode $a$ as 

$$|\Psi^\pm\rangle = (|0\rangle_a + \lambda_1|\pm\rangle_a - \lambda_1|\mp\rangle_a)/\sqrt{2}, \quad (13)$$

$$|\Phi^\pm\rangle = (|0\rangle_a - \lambda_1|\pm\rangle_a + \lambda_1|\mp\rangle_a)/\sqrt{2}, \quad (14)$$

Eq. (12) can be expanded in this basis as 

$$|\Psi\rangle_T = (C_1|0\rangle_b - C_2|1\rangle_b)|\Psi^\ast\rangle + (C_1|0\rangle_b + C_2|1\rangle_b)|\Psi^-\rangle - (C_1|1\rangle_b - C_2|0\rangle_b)|\Phi^\ast\rangle - (C_1|1\rangle_b + C_2|0\rangle_b)|\Phi^-\rangle)/2. \quad (15)$$

C. Bell measurement

Finally, to complete the teleportation process we have to perform a joint measurement on the state $|\Psi\rangle_T$ involving both atom $A_1$ and cavity mode $a$ as represented by one of the four results: $|\Psi^\ast\rangle$, $|\Psi^-\rangle$, $|\Phi^\ast\rangle$, or $|\Phi^-\rangle$. To this end, we note that after atom $A_1$ interacting off resonantly with mode $b$ and being tuned via Stark effect to the classical field, as indicated by Eqs. (4) and (6), respectively, the following result is obtained:

$$|\Psi^\pm\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle_a \pm |1\rangle_a)|e_1\rangle, \quad (16)$$

$$|\Phi^\pm\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle_a \pm |1\rangle_a)|g_1\rangle. \quad (17)$$

Just after, atom $A_1$ is detuned from the classical field to a large negative value which freezes the atom-field evolution. Then measuring atom $A_1$ allows one to distinguish the states $|\Psi^\pm\rangle$ and $|\Phi^\pm\rangle$. Figure 1 shows the steps of atom $A_1$ in the temporal region $[\tau_1, \tau_2]$.

Next, we have to discern the phase ($\pm$). This is done by sending a second atom, $A_2$, in the ground state $|g_2\rangle$ resonantly with mode $a$. This atom, after interacting resonantly with mode $a$, as indicated by Eq. (5), is tuned to the classical field and evolves according to Eq. (4). The result is

$$(0\rangle_a \pm |1\rangle_a)\phi_2 \rightarrow \begin{cases} |0\rangle_a\phi_2 & \text{if (+)} \\ |0\rangle_a\phi_2 & \text{if (−)} \end{cases}. \quad (18)$$

Finally atom $A_2$ is detuned from the classical field to a large negative value, freezing the atom-field evolution. Therefore, when atom $A_2$ is measured in ground state the phase is (+); otherwise the phase is (−). We note from Eq. (15) that the only prompt result that completes the teleportation process from cavity mode $a$ to cavity mode $b$ is a joint measurement resulting in state $|\Psi^\ast\rangle$. The steps suffered by the second atom $A_2$ are shown in the temporal region $[\tau_2, \tau_3]$ of Fig.1.

If the joint measurement results in $|\Psi^\ast\rangle$, a third atom, $A_3$, interacting dispersively with mode $a$ (or mode $b$), can be adjusted to repair the phase: $U_{ab}(3, b)(C_1|0\rangle_b - C_2|1\rangle_b)|e_3\rangle \rightarrow (C_1|0\rangle_b + C_2|1\rangle_b)|e_3\rangle$, the atom $|e_3\rangle$ being dropped. To joint measurements leading to states $|\Phi^\pm\rangle$, the teleportation process cannot be completed unless additional cavities and/or atoms are introduced, thus overcomplicating the scheme. The success probability is then limited to 50%.

IV. CONCLUSIONS

All experiments in cavity QED have been performed, until now, using a single cavity and, although implemented in distinct scenarios, teleportation remains a challenge in cavity QED. This challenge is mainly due to difficulties coming from decoherence of the qubits and from complexities to control interactions in more than one cavity. Here we have presented an oversimplified scheme to teleport a superposition of zero- and one-photon state from one mode to another of a single bimodal high-$Q$ cavity. The two modes have distinct frequencies and orthogonal polarizations, constituting
an application of Ref. [16]. Our scheme is based on the following operations: (i) control of the interaction times between two-level atoms and each one of the two modes sustained by the bimodal high-$Q$ cavity; (ii) use of Stark shifts and classical fields (Ramsey zones); (iii) selective atomic state detectors. Assuming the time spent in experiments is less than typical damping times for the qubits (same order of decoherence time) we have neglected both atomic spontaneous decay and losses in the high-$Q$ cavity. Although our scheme is able to discern each one of the four Bell states, the success probability is limited to 50%. By dropping the use of the Bell-state measurement, our proposal can also be realized using only a single atom and a single bimodal high-$Q$ cavity. However, this alternative reduces the fidelity of the teleported state [14]. We also mention that this scheme is the most economical one used to teleport a superposed state from one mode to another in a cavity, since it requires a single cavity and circular two-level Rydberg atoms, thus opening the way to implement teleportation also in the QED domain. Finally, to discuss the feasibility of our procedure, we consider the typical experimental values of the parameters for Rydberg atoms with principal quantum numbers 50 and 51. This implies the coupling constant $g = 2 \pi \times 47 \text{ kHz}$ and photon damping times $T_{Mb} = 1 \text{ ms}$ and $T_{Mb} = 0.9 \text{ ms}$ [16]. As shown in Fig. 1, atom $A_1$ interacts first resonantly with $M_a$ for a time $\tau_{Ma} = \pi / 2g \approx 5.5 \mu s$; after that it is tuned with classical field (CF) for a time $\tau_{CF} \approx 6.3 \mu s$; in sequence it interacts resonantly with $M_b$ for a time $\tau_{Mb} = \pi / 2g \approx 5.3 \mu s$; next, $A_1$ interacts with CF during the time $\tau_{CF} \approx 3.1 \mu s$, dispersively with $M_b$ for a time $\tau_{dp} = \pi \delta / g^2 \approx 106.4 \mu s$ and again with CF for a time $\tau_{CF} \approx 6.3 \mu s$. So, the total interaction time of $A_1$ results $\approx 132.7 \mu s$ and $\approx 11.6 \mu s$ for $A_2$. These data plus the total time of Stark effect $\approx 8 \mu s$ lead to the total interaction times about 152.3 $\mu s$, lesser than the photon damping times. Hence, the scheme is experimentally feasible within the microwave domain.

**ACKNOWLEDGMENTS**

We thank the VPG/Vice-Reitoria de Pós-Graduação e Pesquisa-UCG (NGA), CAPES (GP), CNPq (NGA, ATA, BB), FUNAPE-GO (BB), Brazilian agencies, for the partial support.