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Resonance generation of photons from vacuum in cavities due to strong periodical changes of conductivity in a thin semiconductor boundary layer

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Abstract
We study a possibility of photon generation from vacuum in a cavity with an artificial effective time-dependent plasma mirror, which could be created by means of periodical short laser pulses, illuminating a thin semiconductor slab. We take into account two important circumstances: a big imaginary part of the complex time-dependent dielectric permeability inside the slab and a strong dependence of this imaginary part on the distance from the surface of the slab. We find the conditions under which the usual unitary quantization schemes in time-dependent media with real dielectric permeability can be applied to the problem concerned with relatively small (a few per cent) error. We show that, by using a slab with thickness of the order of 1 mm, it is possible to generate a large number of microwave (GHz) photons (up to 10^8 or more) after several thousand picosecond pulses with repetition frequency of the order of 1 GHz, provided that semiconductor materials with high mobility of carriers, high photoabsorption efficiency and small recombination time (less than 1 ns) can be found. We discuss the possible advantages of modes with TM polarization over TE ones, as well as some other important problems to be solved.

Keywords: nonstationary Casimir effect, cavities with periodically moving boundaries, thin semiconductor inhomogeneous slab, time-dependent complex dielectric permeability, TE and TM polarizations, Bessel functions, parametric resonance

1. Introduction

The possibility of a significant amplification of classical electromagnetic fields inside cavities with oscillating boundaries under the conditions of parametric resonance seems to have been pointed out for the first time by Askar'yan in 1962 [1], while detailed studies were performed by Krasil'nikov and Pankratov [2, 3]. The dynamics of *quantum* fields in cavities with moving boundaries was considered for the first time by Moore [4], who showed the possibility of photon creation *from vacuum*, noting, however, that this effect should be very small, if the velocities of boundaries are much less than the velocity of light. The possibility of enhancement of *vacuum* (zero point) fluctuations under the conditions of resonance between field modes and oscillations of boundaries was discussed in [5, 6]. However, the first evaluations of the effect gave extremely big and unrealistic numbers, for two reasons: (i) approximate perturbative approaches, used in those papers, are invalid, as a matter of fact, under resonance conditions; (ii) the chosen amplitudes of oscillations of the cavity length were many orders of magnitude bigger than those which could be actually achieved in practice.

Correct calculations were done in the 1990s by different authors in the frameworks of different approaches [7–22] (for an extensive list of publications until 2001 see [23]). They showed that, indeed, a significant amount of photons could be created from vacuum, if the boundaries of a high-Q cavity perform small oscillations at a frequency which is a multiple of some cavity eigenfrequency. This could be one of manifestations of the *nonstationary* [6, 24] (or dynamical [25, 26]) *Casimir effect*. Now, the focus of research has gradually drifted to studies of more or less realistic concrete schemes (instead of simple models considered earlier), which could permit one to observe the effect in a laboratory.

One of the most important parameters, which determines the possible number of created photons, is the achievable value of the wall displacement amplitude. For cavity dimensions of the order of 1–100 cm, the field resonance frequencies $(\omega_0/2\pi)$ belong to the band from 30 GHz to 300 MHz. It is difficult to imagine that the wall could be forced to oscillate as a whole at such a high frequency. The first idea [10, 11] was to excite oscillations of the *surface* of the cavity wall. In such a case, the amplitude a of a standing acoustic wave at frequency $\omega_w = 2\omega_0$ (coinciding with the amplitude of oscillations of the free surface) is related to the relative deformation amplitude δ inside the wall as $\delta = \omega_w a/v_s$, where v_s is the velocity of sound. Since usual materials cannot bear deformations exceeding the value $\delta_{max} \sim 10^{-2}$, the maximal possible velocity of the boundary is $v_{max} \sim \delta_{max} v_s \sim 50$ m s^{-1} (independent of the frequency). The maximal relative displacement $\varepsilon = a/L_0$ is $\varepsilon_{max} \sim (v_s/2\pi c)\delta_{max} \sim 3\times10^{-8}$ for the lowest mode with the frequency $\omega_0 \sim c\pi/L_0$. The mean number of photons created from vacuum in a three-dimensional cavity in the resonance mode with frequency ω_0 is given by the formula [10]

$$\langle n\rangle(t) = \sinh^2(\varepsilon\omega_0 t\zeta/2), \tag{1}$$

where $\zeta < 1$ is a numerical coefficient, which depends on the cavity geometry. Then, taking $\varepsilon = 10^{-9}$, $\omega_0/(2\pi) = 10$ GHz, $\zeta = 1/3$, in $t = 1$ s one could get a large number $\sinh^2(10) \sim 10^8$ photons in an empty cavity. However, in order to accumulate photons during 1 s without losses, one needs a cavity with a Q-factor of the order of 10^{10}. Moreover, it is necessary to maintain the resonance condition, which means that the frequency of the wall oscillations must not deviate from $2\omega_0/(2\pi)$ by more than [17] $\delta\omega/(2\pi) < \varepsilon\omega_0\zeta/(2\pi) \sim 3$ Hz during the 1 s. Until now, it is unclear how to excite strong surface vibrations with frequencies in GHz range, and no concrete schemes were proposed.

On the other hand, analogies and differences between the moving ideal mirror problem and problems of moving dielectrics or media with time-dependent parameters have also been discussed for the past 15 years by many authors [27–42]. In particular, an idea to create an *effective* electron–hole 'plasma mirror' inside a semiconductor slab, illuminated by a powerful femtosecond laser pulse, was proposed in [43]. Experimental work in this direction is now going on [44]. It is planned to simulate *periodical* displacements of a mirror, irradiating a thin semiconductor slab on one surface of a cavity by periodical laser pulses. Since the possibility of fast transformation of a semiconductor slab to a highly conducting one has been demonstrated experimentally [45], an effective length of the cavity can be changed by the quantity, which has an order of magnitude of the thickness of the semiconductor slab (standard GaAs or Si plates have thickness from 600 μm to 1 mm). Therefore, a relative change of the field mode eigenfrequency can be made several orders of magnitude bigger than in the 'acoustical' scheme. As a consequence, one can reduce in the same proportion the number of oscillations of the boundary necessary to produce photons in the cavity, thus relaxing the requirements for the Q-factor of the cavity and the admissible detuning from the exact resonance. Another advantage is the possibility to use periodic pulses of arbitrary shape, whose period can be much longer than the period of oscillations of the chosen mode of electromagnetic field.

In view of the new scheme, it is necessary to analyse carefully its strong and weak sides, trying to discover possible 'underwater rocks', which could destroy the experiment. The first steps in this direction were made in recent publications [46, 47], where two different models of time-dependent dielectric slabs were considered. In [46], a thin dielectric layer was replaced (following [33]) by a time-dependent effective δ-potential, whereas the authors of [47] considered a thin dielectric slab with time-dependent permeability, which did not vary across the slab. Here we make the next step, trying to take into account two important circumstances.

The first one is that the dielectric permeability $\varepsilon(x)$ is a *complex function*: $\varepsilon = \varepsilon_1 + i\varepsilon_2$, where

$$\varepsilon_2 = 2\sigma/f, \tag{2}$$

σ and f being the conductivity inside the semiconductor slab (in CGS units) and the frequency in Hz, respectively. For example, for Cu at 2.5 GHz, $\varepsilon_2 \sim 10^8$. On the other hand, all existing models of photon generation in cavities with time-dependent parameters are based on the assumption that ε is a *real* quantity, and the quantum evolution is *unitary* (see section 2 for details). Using simplified models, people avoid this difficulty, either neglecting the imaginary part of ε, or considering the limiting case of very high conductivity (ideal mirrors). Unfortunately, this cannot be done in the case concerned. Although the conductivity is, indeed, very small for a non-illuminated semiconductor slab at low temperatures, and it can be very high after illumination (when high concentrations of carriers are achieved), it inevitably assumes intermediate values in the process of excitation, when ε_2 gradually increases from very low to very high values, so that for some time interval it becomes of the same order of magnitude as ε_1. This means that, strictly speaking, existing quantization methods must be revised and generalized, in order to describe correctly the results of planned experiments. However, because such generalizations are not available yet, we consider a moderate task: to try to find such regimes of excitation, where the complex nature of ε would not influence significantly the rate of photon generation.

Another important circumstance, which must be taken into account, is that the imaginary part of the dielectric permeability ε_2 rapidly decreases with increase of distance from the illuminated surface inside a semiconductor slab, because the conductivity is proportional to the concentration of carriers, which are supposed to be created due to absorption of laser radiation inside the semiconductor. Obviously, the laser intensity decreases monotonically inside the medium. It is clear that the absorption must be strong enough, otherwise an efficient plasma mirror will not be created; moreover, the unabsorbed part of the radiation will heat the cavity walls and cause many other undesired effects. Consequently, models based on the assumption of homogeneous dielectric permeability ε across the slab can be rather far from reality in the case concerned. Therefore, we consider a somewhat more adequate model, choosing some prescribed reasonable dependence of ε_2 on the longitudinal coordinate (in the direction perpendicular to the surface of slab).

The plan of the paper is as follows. In section 2 we bring together the main results of the existing theories, describing the process of photon generation in ideal cavities with time-dependent parameters. In section 3 we calculate the resonance frequency shift of the cavity due to the presence of a thin slab with inhomogeneous dielectric permeability of a special form, considering the special case of TE polarization of the field (the Dirichlet boundary conditions). In section 4 we 'turn on' the time dependence of the dielectric function (also of some special kind, which seems reasonable at least qualitatively). Comparing the values of the single pulse 'reflection coefficient' from an effective time-dependent 'potential barrier' (which determines the rate of photon generation), obtained with the aid of total complex frequency shift and its real part, we find the domains of parameters where the results of calculations are close. We interpret the coincidence of results as a justification for neglecting the nonunitarity of evolution and using standard quantization methods. Section 5 is devoted to the discussion of results obtained. In the appendix we consider briefly the effects of polarization of the field, showing that TM polarization has some advantages over the TE one for an arbitrary law of motion of a flat boundary of a 3D cavity.

2. Generation of photons from vacuum due to small periodical changes of cavity parameters: general relations

Our approach is based on the method of *effective Hamiltonian*, proposed in [32] and developed by other authors, especially in the papers [47, 48] (for other references see [23]). In brief, the scheme is as follows. Suppose that the set of Maxwell's equations in a medium with *time-independent* parameters and boundaries can be reduced to an equation of the form

$$\hat{\mathcal{K}}(\{L\})\mathbf{F}_\alpha(\mathbf{r};\{L\}) = \omega_\alpha^2(\{L\})\mathbf{F}_\alpha(\mathbf{r};\{L\}), \tag{3}$$

where $\{L\}$ means a set of parameters, including, for example, the distance L between the walls, $\omega_\alpha(\{L\})$ is the eigenfrequency of the field mode, labelled by the number (or a set of numbers) α, and $\mathbf{F}_\alpha(\mathbf{r};\{L\})$ is some (in general, vector) function, whose knowledge enables one to calculate all components of the electromagnetic field (e.g., the vector potential, dual potential, Herz vector, etc). In the simplest cases, equation (3) is reduced to the Helmholtz equation, and the operator $\hat{\mathcal{K}}(\{L\})$ is reduced to the Laplace operator. Usually, the operator $\hat{\mathcal{K}}(\{L\})$ is self-adjoint, and the set of functions $\{\mathbf{F}_\alpha(\mathbf{r};\{L\})\}$ is orthonormalized and complete in some sense.

Now suppose that the parameters $\{L\}$ become time dependent (for example, a part of the boundary is a plane surface moving according to a *prescribed* law of motion $L(t)$). If one can still satisfy automatically the boundary conditions, expanding the field $\mathbf{F}(\mathbf{r}, t)$ over 'instantaneous' eigenfunctions,

$$\mathbf{F}(\mathbf{r}, t) = \sum_\alpha q_\alpha(t)\mathbf{F}_\alpha(\mathbf{r};\{L(t)\}) \tag{4}$$

(this is true, e.g., for the Dirichlet boundary conditions, which are equivalent in some cases to the TE polarization of the field modes; for a more complicated case of TM polarization and Neumann's conditions see, e.g., [49]), then the dynamics of the field is described completely by the generalized coordinates $q_\alpha(t)$, whose equations of motion can be derived from the *effective time-dependent Hamiltonian* [48]

$$H = \frac{1}{2}\sum_\alpha \left[p_\alpha^2 + \omega_\alpha^2(L(t))q_\alpha^2\right] + \frac{\dot{L}(t)}{L(t)}\sum_{\alpha\neq\beta} p_\alpha m_{\alpha\beta} q_\beta, \tag{5}$$

$$m_{\alpha\beta} = -m_{\beta\alpha} = L\int \mathrm{d}V\,\frac{\partial \mathbf{F}_\alpha(\mathbf{r};L)}{\partial L}\mathbf{F}_\beta(\mathbf{r};L). \tag{6}$$

In a generic case of arbitrary time-dependent frequencies ω_α and coefficients $m_{\alpha\beta}$, finding solutions of the Schrödinger or Heisenberg equations corresponding to the Hamiltonian (5) which hold *in the long-time limit* is a difficult problem. However, there exist important exceptions. Namely, in the case of a single space dimension (the Fabry–Perot resonator) and *small harmonical* oscillations of boundaries at some *resonance* frequencies, it is possible to reduce the Heisenberg equations of motion to a simple set of equations which can be solved analytically [11, 16]. This happens due to an *equidistant* form of the spectrum of field eigenfrequencies and a specific form of coefficients $m_{\alpha\beta}$.

On the other hand, the eigenfrequency spectra of *three-dimensional* cavities (which one deals with in real life) are *non-equidistant*. Due to this fact, *under the resonance conditions*, different modes practically do not interact *in the long-time limit*, and one can consider only *single resonance mode*. Thus the problem is reduced to a simple model of a *one-dimensional quantum oscillator with a time-dependent frequency* $\omega(t)$, which is determined by an *instantaneous geometry* of the cavity [10, 11]. It is worth emphasizing important points, which led us to this conclusion.

(i) An absence of 'accidental equidistant parts' in the spectrum. Such parts can arise due to some geometrical symmetries (for example, in cubical cavities) [50]. In this case one arrives at a model of *two coupled oscillators* with a specific 'coordinate-momentum' coupling, studied in [50, 51].

(ii) The Dirichlet boundary conditions (or TE polarization of the field [49]) and a possibility of expansion (4). In more complicated situations (such as, for example, generalized Neumann's conditions for TM modes in the case of moving boundaries [49]), the effective frequency is not determined by an instantaneous geometry, but it depends of the history of process: see the appendix.

(iii) An ‘uncoupling’ of different modes was demonstrated in [10, 11] for the *harmonical* motion of the boundary. Below we *assume* that the interaction between different modes can be neglected for more or less arbitrary periodical motions of boundary. Although such an assumption seems reasonable from the point of view of physical intuition, we have no proof. This point needs further investigation (actually, we suppose that all higher harmonics of different modes are out of resonance), especially because the period of the wall’s oscillations can be shifted from a multiple period of the field oscillations in the selected mode (see equation (13) below).

Having in mind these remarks, we assume that the one-dimensional Hamiltonian (5) with some time-dependent effective frequency can serve as a reasonable model for the description of the process of photon creation from vacuum in a three-dimensional nondegenerate cavity with time-dependent parameters. The Schrödinger equation with such a Hamiltonian was solved for the first time in the seminal paper by Husimi [52] in 1953, and since that time, the problem of a quantum oscillator with time-dependent parameter has been studied in detail in numerous publications (for the most recent review see, e.g., [53]).

One of the main results of these studies is that all dynamical properties of the *quantum* oscillator are determined by the fundamental set of solutions of the *classical* equation of motion

$$\ddot{x} + \omega^2(t)x = 0. \tag{7}$$

In particular, if $\omega(t) = \omega_{\rm i}$ for $t \to -\infty$ and $\omega(t) = \omega_{\rm f}$ for $t \to \infty$ (i.e., all parameters of the cavity, such as the wall positions or the dielectric permeability in each point, assume some fixed values before and after an experiment), then the information on the final state of the quantum oscillator, which was initially in the vacuum state, is encrypted in the complex coefficients $\rho_\pm$ of the asymptotical form of solutions of equation (7),

$$x_{t\to\infty} = \omega_{\rm f}^{-1/2}\left[\rho_- {\rm e}^{-{\rm i}\omega_{\rm f}t} + \rho_+ {\rm e}^{{\rm i}\omega_{\rm f}t}\right], \tag{8}$$

satisfying the initial condition $x_{t\to-\infty} = \omega_{\rm i}^{-1/2}{\rm e}^{-{\rm i}\omega_{\rm i}t}$. The mean number of quanta for $t \to \infty$ can be expressed in terms of $\rho_\pm$ as [30, 54]

$$\mathcal{N} = |\rho_+|^2 = R/T, \tag{9}$$

where the quantities

$$R \equiv |\rho_+/\rho_-|^2, \qquad T \equiv 1 - R \equiv |\rho_-|^{-2} \tag{10}$$

can be interpreted as energy reflection and transmission coefficients from an effective ‘potential barrier’ $\omega^2(t)$, and the identity $|\rho_-|^2 - |\rho_+|^2 = 1$ holds due to the unitarity of evolution.

A simple consequence of formula (9) is that no photons can be created during a single pass of the wall from one position to another, because for *monotonic* functions $\omega(t)$ the reflection coefficient is limited [55] by the Fresnel formula:

$$R \leqslant R_{\rm F} = \left(\frac{\omega_{\rm i} - \omega_{\rm f}}{\omega_{\rm i} + \omega_{\rm f}}\right)^2, \tag{11}$$

and the equality can be achieved only in the case of very rapid change of position (for a time much less than the period of field oscillations). If the relative change of the effective frequency is small, then obviously $R_{\rm F} \ll 1$. Moreover, in real life $R_{\rm F} \ll 1$, even if the change of frequencies is large, because real boundaries in laboratory experiments can only move with velocities much less than the speed of light, so that the process is almost adiabatic.

However, it is well known that the reflection coefficient can be made very close to unity in the case of *periodical* variations of parameters, due to interference. For a periodical motion of the wall, the function $\omega^2(t)$ has the form of a periodical sequence of pulses, which can be considered as effective ‘potential barriers’. If pulses are separated by equal intervals of time, when $\omega = \omega_0 =$ constant, then the dynamics can be completely characterized by two complex amplitude reflection and transmission coefficients of each ‘barrier’, r and f^{-1}, and the phase shift $\theta \equiv \omega_0 T$, where T is the period between pulses. Using the standard method of *transfer matrix*, the following expression for the mean number of photons, created after n pulses, can be obtained [56]:

$$\mathcal{N}_n = |rf|^2 \frac{\sinh^2(n\nu)}{\sinh^2(\nu)}, \tag{12}$$

where

$$\cosh\nu \equiv \pm\,{\rm Re}\left(f{\rm e}^{{\rm i}\theta}\right). \qquad f \equiv |f|{\rm e}^{{\rm i}\varphi}$$

(Note that equation (8) is complex conjugated with respect to that considered in [56], and the same is true for the inverse transmission coefficient f; therefore the phases φ used here and in [56] have opposite signs.) A generation of photons is possible, if parameter ν is real, i.e., $|f|\,|\cos(\varphi + \theta)| > 1$. Since $|f| > 1$, one has to adjust the phase shift θ to the phase φ of the inverse transmission coefficient. The maximal effect is achieved for $\theta = \pi m - \varphi$ (with m an integer). This is equivalent to the following relation between the periodicity of pulses $T = \theta/\omega_0$, the period of the electromagnetic field oscillations $T_0 = 2\pi/\omega_0$ and the phase φ:

$$T = \tfrac{1}{2}T_0\,(m - \varphi/\pi)\,. \tag{13}$$

For anharmonic oscillations of the boundary (or other characteristics of the cavity), the field mode can be excited not only under the condition of the parametric resonance $T = T_0/2$, but the period of motion of the boundary can be greater than the period of field oscillations. Under the condition (13), $\cosh(\nu) = |f|$ and $\sinh(\nu) = |rf|$, so that

$$\mathcal{N}_n^{\rm (res)} = \sinh^2(n\nu)\,. \tag{14}$$

Consequently, a concrete form of the function $\omega(t)$ is not very important for the exponential dependence of the number of created quanta on the number of wall’s oscillations n, if $n\nu \gg 1$ (although it influences, of course, the concrete value of the coefficient ν). What is important is the fulfilment of the *resonance condition* (13).

To tune the system in resonance, one has to know the concrete value of the phase φ in (13). For small values of $|r| \ll 1$, the critical condition $|f|\cos(\Delta\theta_{\rm c}) = 1$ becomes $|\Delta\theta_{\rm c}| = |rf| \approx |r|$. Consequently, admissible deviations of the periodicity of pulses from the resonance value (13) must not exceed the quantity $(\Delta T)_c = |r|/(2\pi)$. This implies that the phase φ must be known with an accuracy better than $(\delta\varphi)_{\max} = |r|$.

For small variations of the effective frequency $\omega(t)$, one can express, following [57], the absolute value of amplitude reflection coefficient as

$$r \approx \frac{1}{2}\left|\int_{-\infty}^{\infty} \frac{\dot{\omega}(t)}{\omega(t)} \exp\left[-2\mathrm{i}\int_{t_0}^{t} \omega(\tau)\,\mathrm{d}\tau\right] \mathrm{d}t\right|.$$

Writing

$$\omega(t) = \omega_0[1 + \chi(t)] \tag{15}$$

with $|\chi| \ll 1$, we obtain

$$|r| \approx \left|\int_{-\infty}^{\infty} \frac{\mathrm{d}\chi}{\mathrm{d}t}\, \mathrm{e}^{-2\mathrm{i}\omega_0 t}\, \frac{\mathrm{d}t}{2}\right| = \left|\int_{-\infty}^{\infty} \omega_0 \chi(t) \mathrm{e}^{-2\mathrm{i}\omega_0 t}\, \mathrm{d}t\right| \tag{16}$$

for the absolute value of the effective amplitude reflection coefficient from the single barrier and

$$\varphi \approx \omega_0 \int_{-\infty}^{\infty} \chi(t)\, \mathrm{d}t \tag{17}$$

for the phase of the single-barrier inverse transmission coefficient.

Comparing equations (9) and (14), one can see that under the resonance conditions, the absolute value of the total (amplitude) reflection coefficient from n identical barriers, $|r_n|$, is given by the formula $|r_n| = \tanh(n|r_1|)$. It is interesting to notice in this connection, that applying equation (16) formally to n equidistant barriers in the case of resonance (when contributions of all barriers to the integral (16) have identical phase factors), one would obtain the result $\tilde{r}_n = n\tilde{r}_1$, where the tilde means that one uses the approximate integral formula (16). On the other hand, according to [57], for large values of $\tilde{r}$ one should use an interpolation formula $|r| = \tanh|\tilde{r}|$, obtained in [58]. We see that under the resonance conditions, this interpolation formula gives practically exact results for any number of equidistant barriers (provided $|\tilde{r}_1| \ll 1$).

3. Resonance frequency shift of an empty cavity with a thin inhomogeneous semiconductor slab

We consider a cylindrical cavity with an arbitrary cross section and the axis parallel to the x-direction. We assume that the main part of the cavity is empty: $\varepsilon(x) \equiv 1$ for $-L < x < 0$, but there is a thin slab of a semiconductor material with $\varepsilon(x) \neq 1$ in the region $0 < x < D \ll L$.

The calculation of an exact profile of the function $\varepsilon_2(x)$ (the imaginary part of the complex dielectric permeability), arising in the process of photoabsorption of a laser pulse inside the slab, is a complicated problem, especially when the concentration of carriers becomes high. We confine ourselves to the simplest model, which seems reasonable at least qualitatively. Namely, we suppose that the function $\varepsilon_2(x)$ has the exponential form,

$$\varepsilon_2(x) = B\exp(-\gamma x), \tag{18}$$

where γ is a constant absorption coefficient of laser radiation inside the semiconductor, and the coefficient B (which is proportional to the intensity of the laser pulse) depends on time as a parameter. Since the absorption must be significant, we consider the case when the product γD is not very small: $\gamma D \sim 1$ or $\gamma D > 1$, so that $\gamma L \gg 1$ in any case. As for the real part ε_1 of the complex dielectric function, we assume that it is constant.

To obtain a concrete form of equation (3), which determines the spectrum of eigenfrequencies, we start from the Maxwell equations in the form which does not contain derivatives of the function $\varepsilon(\mathbf{r}, t)$ with respect to the time variable (as soon as time in Hamiltonian (5) is considered as a parameter):

$$\mathrm{rot}\,\mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \qquad \mathrm{rot}\,\frac{\mathbf{D}}{\varepsilon} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{19}$$

(the medium is assumed to be non-magnetic). Excluding the magnetic field $\mathbf{B}$, we obtain the equation

$$\mathrm{rotrot}\,\frac{\mathbf{D}}{\varepsilon} = -\frac{1}{c^2}\frac{\partial^2 \mathbf{D}}{\partial t^2} \tag{20}$$

or

$$\mathrm{graddiv}\,\frac{\mathbf{D}}{\varepsilon} - \Delta\frac{\mathbf{D}}{\varepsilon} = \frac{\omega^2}{c^2}\mathbf{D} \tag{21}$$

for the monochromatic field $\mathbf{D}(\mathbf{r}, t) = \mathbf{D}(\mathbf{r})\exp(-\mathrm{i}\omega t)$.

In the case of TE modes, vectors $\mathbf{E}$ and $\mathbf{D}$ are perpendicular to the x-axis (and parallel to the plane surfaces of the cylinder and slab). Since we assume that the dielectric permeability depends only on the longitudinal space variable x, the first term in equation (21) is zero:

$$\mathrm{div}\,\frac{\mathbf{D}}{\varepsilon} = \frac{1}{\varepsilon(x)}\,\mathrm{div}\,\mathbf{D} \equiv 0,$$

because all differentiations are made in this case with respect to transversal coordinates, which function $\varepsilon(x)$ does not depend on. Consequently, the electric field $\mathbf{E} = \mathbf{D}/\varepsilon(x)$ satisfies the usual three-dimensional Helmholtz equation

$$\Delta\mathbf{E} + (\omega/c)^2\varepsilon(x)\mathbf{E} = 0. \tag{22}$$

It is known that equation (22) allows for a factorization of any scalar component of the electric field $E(x, \mathbf{r}_\perp)$ (where $\mathbf{r}_\perp$ stands for the coordinates in the plane perpendicular to the x-axis, and the choice of scalar components depends on a concrete form of the cavity): $E(x, \mathbf{r}_\perp) = \psi(x)\Phi(\mathbf{r}_\perp)$, where the function $\Phi(\mathbf{r}_\perp)$ obeys the two-dimensional Helmholtz equation

$$\Delta_\perp\Phi + k_\perp^2\Phi = 0. \tag{23}$$

Thus the equation for $\psi(x)$ becomes

$$\psi'' + \left[(\omega/c)^2\varepsilon(x) - k_\perp^2\right]\psi = 0. \tag{24}$$

Its solution in the region $-L < x < 0$, satisfying the boundary condition $\psi(-L) = 0$, is obvious:

$$\psi(x) = F_1\sin[k(x + L)], \tag{25}$$

where the constant coefficient k is related to the field eigenfrequency ω and the corresponding wavelength in vacuum λ as

$$\omega = c\left(k^2 + k_\perp^2\right)^{1/2}, \qquad \lambda = 2\pi\left(k^2 + k_\perp^2\right)^{-1/2}. \tag{26}$$

To solve equation (24) in the region $0 < x < D$, where $\varepsilon(x) = \varepsilon_1 + \mathrm{i}B\exp(-\gamma x)$, we introduce a new variable

$$z = z_0 \exp(-\gamma x/2), \qquad z_0 = \frac{4\pi\sqrt{\mathrm{i}B}}{\lambda\gamma}, \tag{27}$$

and arrive at the *Bessel equation* with complex index and complex argument:

$$z^2\psi_{zz} + z\psi_z + (\nu^2 + z^2)\psi = 0, \tag{28}$$

$$\nu = \frac{2}{\gamma}\sqrt{k^2\varepsilon_1 + k_\perp^2(\varepsilon_1 - 1)}. \tag{29}$$

The solution to (28), satisfying the condition $\psi(z_D) = 0$, $z_D = z_0\exp(-\gamma D/2)$, can be written as (we omit the normalization constant)

$$\psi(z) = J_{i\nu}(z)\, J_{-i\nu}(z_D) - J_{i\nu}(z_D)\, J_{-i\nu}(z). \tag{30}$$

Since functions (25) and (30) must assume the same values at $x = 0$, as well as their derivatives, we arrive at the transcendental equation, which determines the value of the coefficient k:

$$\tan(kL) = \frac{2k}{z_0\gamma}\,\frac{J_{-i\nu}(z_0)\,J_{i\nu}(z_D) - J_{i\nu}(z_0)\,J_{-i\nu}(z_D)}{J'_{i\nu}(z_0)\,J_{-i\nu}(z_D) - J'_{-i\nu}(z_0)\,J_{i\nu}(z_D)}, \tag{31}$$

where $J'_\nu(z)$ is the derivative of function $J_\nu(z)$ with respect to its argument.

Equation (31) can be solved numerically, but we prefer to obtain an approximate analytical solution, taking into account that under the condition $D \ll L$, the value of k must be close to π/L. Thus we write

$$k = (1 + \xi)\pi/L, \qquad |\xi| \ll 1$$

and replace the left-hand side of (31) by $\tan(\pi\xi) \approx \pi\xi$, whereas in the right-hand side we identify k with π/L (as soon as the small parameter D/L appears there). Moreover, we take into account that the absolute value of index ν is very small, being of the order of $4\pi\sqrt{\varepsilon_1}/(\gamma\lambda) \ll 1$. Then, using the integral representation of Bessel functions

$$\Gamma(\nu + 1/2)J_\nu(z) = \frac{2}{\sqrt{\pi}}\left(\frac{z}{2}\right)^\nu \int_0^{\pi/2} \cos(z\sin\varphi)(\cos\varphi)^{2\nu}\,\mathrm{d}\varphi$$

and replacing ν by zero in the argument of the Gamma function and in the integrand, we arrive at an approximate formula (note that the argument z can be large)

$$J_\nu(z) \approx (z/2)^\nu J_0(z), \qquad |\nu| \ll 1. \tag{32}$$

Putting this formula into equation (31), one can see that odd and even combinations of functions $J_{\pm i\nu}\left(z_0\mathrm{e}^{-\gamma D/2}\right)$ give rise to the functions $\sin(y)$ and $\cos(y)$, respectively, where

$$y = D\left[\varepsilon_1(\pi/L)^2 + (\varepsilon_1 - 1)k_\perp^2\right]^{1/2}.$$

By the order of magnitude, $y \sim D/\min(L, L_\perp) \sim D/\lambda \ll 1$, where $L_\perp = \pi/k_\perp$ is a characteristic transversal dimension of the cavity. (This estimation holds, provided ε_1 is not extremely big and not too close to unity; but this is just the case of typical semiconductors: for example, $\varepsilon_1 = 13.2$ for GaAs.) Therefore, one can replace $\sin(y)$ by y and $\cos(y)$ by 1. Taking into account all these simplifications, as well as the relation $J'_0(z) = -J_1(z)$, we arrive at a simple formula

$$\xi \approx \frac{\eta\Delta J_0\left(4\pi j\alpha\sqrt{B}\right)}{j\pi\Delta\sqrt{B}J_1\left(4\pi j\alpha\sqrt{B}\right) - J_0\left(4\pi j\alpha\sqrt{B}\right)}, \tag{33}$$

where

$$\eta = \frac{\lambda}{2L}, \quad \Delta = \frac{2D}{\lambda}, \quad \alpha = \frac{1}{\lambda\gamma}, \quad j = \exp\left(\mathrm{i}\frac{\pi}{4}\right). \tag{34}$$

Obviously, $\eta < 1$, whereas $\Delta \ll 1$ and $\alpha \ll 1$. In realistic situations, when $\gamma D > 1$, we have $\alpha < \Delta/2$. Note that the real part of the dielectric permeability, ε_1, does not enter the formulae any more.

For $|\xi| \ll 1$, the relative shift of frequency, defined by equation (15), can be written as $\chi = \eta^2(\xi - \xi_0)$, where ξ_0 is the value of ξ for $B = 0$ (i.e., for the nonexcited semiconductor slab). Equation (33) yields $\xi_0 = -\eta\Delta$ (which corresponds to an initial empty cavity of length $L + D$). Consequently,

$$\chi \approx \frac{j\pi\eta^3\Delta^2\sqrt{B}J_1\left(4\pi j\alpha\sqrt{B}\right)}{j\pi\Delta\sqrt{B}J_1\left(4\pi j\alpha\sqrt{B}\right) - J_0\left(4\pi j\alpha\sqrt{B}\right)}. \tag{35}$$

We see that the dependence of the relative frequency shift (and, consequently, the effective reflection coefficient $|r|$ (16)) on the cavity length L, for fixed values of parameters λ (i.e., the resonance frequency), γ (the absorption length) and D (the thickness of semiconductor slab), is very simple: $\chi(\eta) = \eta^3\chi(\eta = 1)$.

The first terms of the Taylor expansion of (35) with respect to $\alpha\sqrt{B}$ are as follows:

$$\chi = 2\eta^3\pi^2\left[-\mathrm{i}B\alpha\Delta^2 + 2B^2\alpha^2\Delta^2(\alpha + \Delta) + \cdots\right]. \tag{36}$$

Consequently, even for large values of B (or conductivity), the imaginary part of the resonance frequency shift can be much greater than the real part, if $\alpha \ll 1$. The expansion (36) indicates that the real part of χ can exceed the absolute value of the imaginary part, provided

$$\mathcal{F} \equiv B\alpha\Delta > 1. \tag{37}$$

On the other hand, using the asymptotical expansion

$$J_\nu(z) \sim (\pi z/2)^{-1/2}\cos[z - (2\nu + 1)\pi/4], \qquad |z| \gg 1,$$

one obtains that

$$\chi = \eta^3\Delta\left[1 - \frac{j}{\pi\Delta\sqrt{B}} + \cdots\right]$$

for $B\alpha^2 > 1$ (note that the value $\chi = \eta^3\Delta$ is equivalent to the displacement of the empty cavity wall from $x = D$ to $x = 0$). In this case, obviously $\mathcal{F} > 1$, as soon as $\Delta > \alpha$. Thus one can ask, what condition can guarantee that $|\,\mathrm{Im}\,\chi| \ll \mathrm{Re}\,\chi$ and $\mathrm{Re}\,\chi$ is close to the asymptotical value $\eta^3\Delta$: the inequality $B\alpha^2 > 1$ or a weaker inequality $B\alpha\Delta > 1$? The answer is given by figure 1, which clearly shows that the sufficient condition is $\mathcal{F} > 1$ (otherwise the horizontal spacing between the curves would be twice as big).

The physical meaning of parameter $\mathcal{F}$ is that it is proportional to the energy of the laser pulse W, or the total

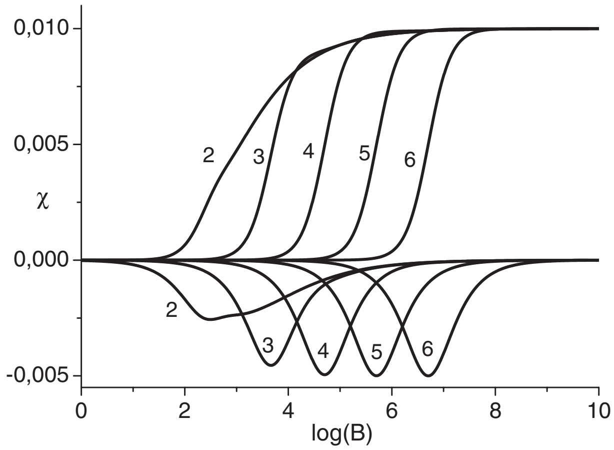

Figure 1. The dependence of the real (upper curves) and imaginary (lower curves) parts of the relative eigenfrequency shift χ (35) on the parameter $\log(B)$, for fixed values $\eta = 1$, $\Delta = 0.01$ and different values of parameter $\alpha = 10^{-k}$, for $k = 2, 3, 4, 5, 6$ (the numbers at each curve give the corresponding values of k).

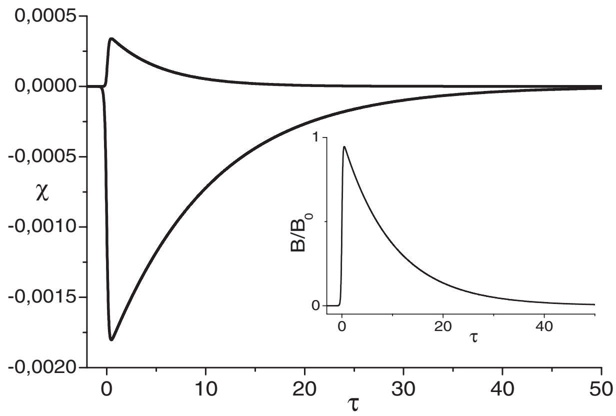

Figure 2. The dependence of the real (upper curve) and imaginary (lower curve) parts of the relative eigenfrequency shift χ (35) on the time variable $\tau = \omega_0 t$, for fixed values of parameters $B_0 = 10^4$, $\mu = 10$, $\beta = 0.1$, $\eta = 1$, $\Delta = 0.01$ and $\alpha = 10^{-4}$. In the inset: the function $B(\tau)/B_0$ (39) for the same values of parameters.

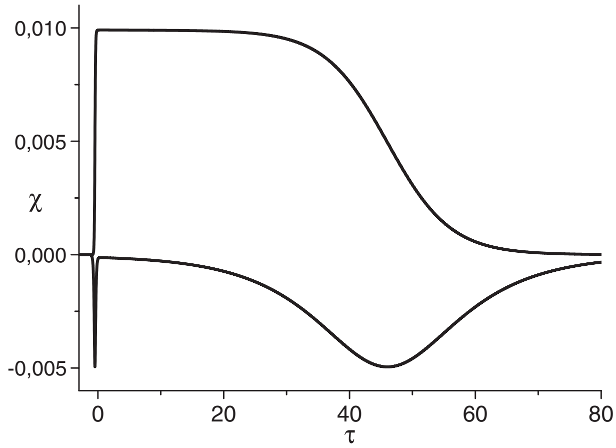

Figure 3. The dependence of the real (upper curve) and imaginary (lower curve) parts of the relative eigenfrequency shift χ (35) on the time variable $\tau = \omega_0 t$, for fixed values of parameters $B_0 = 5 \times 10^6$, $\mu = 10$, $\beta = 0.1$, $\eta = 1$, $\Delta = 0.01$, and $\alpha = 10^{-4}$.

number of carriers created by the pulse. The chain of relations leading to this conclusion is as follows. The laser radiation is absorbed in a slab of the depth $l \sim \gamma^{-1}$ (all estimations here are made by orders of magnitude). It creates $N \sim AW/E_g$ electron–hole pairs, where E_g is the energy gap of the semiconductor material and $A < 1$ is the efficiency of photoabsorption. The characteristic concentration of carriers is $n_e \sim N/lS$, where S is the surface area. The conductivity is $\sigma = n_e e b$, where e is the electron charge and b an average mobility of carriers. Then, using formula (2), we obtain the estimation

$$\mathcal{F} \sim 4A \frac{ebD}{cS\lambda} \frac{W}{E_g}. \tag{38}$$

Taking $A = 1$, $\lambda = 12$ cm, $D = 0.6$ mm, $S = 7 \times 2$ cm^2, $E_g \sim 1.4$ eV (as for GaAs) and $b \sim 3 \times 10^5$ cm^2 V^{-1} s^{-1} $\sim 10^8$ CGS units, we find, as a reference point, that $\mathcal{F} = 1$ for $W \sim 10^{-4}$ J.

4. Single pulse effective 'reflection coefficient'

Now we turn to the nonstationary problem. Again, we consider the most simple reasonable model, supposing that parameter B in equation (18) (i.e., the maximal value of the imaginary part of dielectric constant, achieved at $x = 0$) depends on time as

$$B(t) = \frac{B_0 e^{-t/T_r}}{1 + e^{-t/T_c}} = \frac{B_0 e^{-\beta\tau}}{1 + e^{-\mu\tau}} \equiv B_\tau, \tag{39}$$

$$\tau = \omega_0 t, \qquad \beta = \frac{T_0}{2\pi T_r}, \qquad \mu = \frac{T_0}{2\pi T_c}, \tag{40}$$

where $T_0 = 2\pi/\omega_0$ is the period of field oscillations, whereas parameters T_c and T_r can be interpreted as the time of creation of electron–hole plasma in the slab and the time of recombination, respectively. For the resonance frequency $\nu = 2.5$ GHz, one has $T_0 = 400$ ps. Then, for example, the value $\mu = 10$ corresponds to $T_c \approx 6$ ps, $\mu = 100$ corresponds to $T_c \approx 0.6$ ps, and $\beta = 0.1$ corresponds to $T_r \approx 600$ ps. As for coefficient γ (and the related small parameter α), we assume that they do not depend on time. This can be justified, if the characteristic diffusion time of carriers is much bigger than the recombination time.

Two time dependences of the relative frequency shift $\chi(\tau)$ are shown in figures 2 and 3. Figure 2 corresponds to the case when parameter $\mathcal{F}_0$ (37) (calculated for B_0) is small, $\mathcal{F}_0 = 0.01$. We see that function $\chi(\tau)$ behaves as a scaled function $B(\tau)$, but the imaginary part of χ is much bigger than the real part, so the theory based on the unitary evolution cannot be applied.

Figure 3 corresponds to $\mathcal{F}_0 = 5$. Now, the form of $\chi(\tau)$ has nothing in common with $B(\tau)$, and the imaginary part of χ is close to zero, except for a short initial interval and some relatively short interval at the end of pulse. In this case, formula (16) suggests that one can neglect the imaginary part of χ (at least, in the first approximation), taking the time T_c sufficiently short (or μ big) and the recombination time T_r sufficiently long (β small). Then the slowly varying in time imaginary 'tail' of $\chi(\tau)$ will not contribute to $|r|$ due to the presence of rapidly oscillating exponential $\exp(-2i\tau)$, and the contribution of the initial 'splash' of Im $\chi(\tau)$ will be also small,

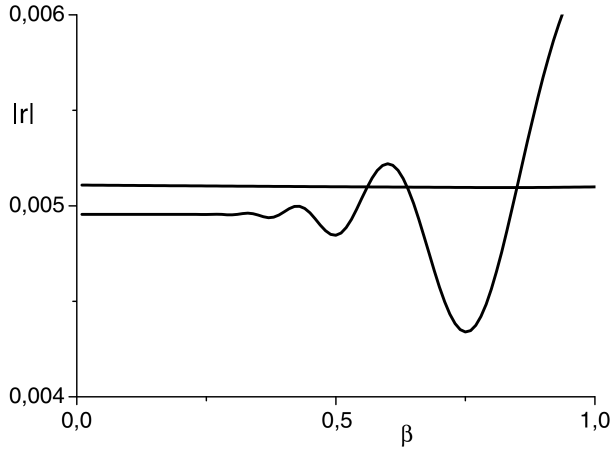

Figure 4. Absolute values of the single pulse effective 'reflection coefficient' $|r|$, calculated by means of the real part of the relative frequency shift $\mathrm{Re}\,\chi(\tau)$ (oscillating curve) and the total complex function $\chi(\tau)$ (straight line), as functions of parameter β, for fixed values of parameters $B_0 = 5 \times 10^6$, $\mu = 100$, $\eta = 1$, $\Delta = 0.01$, and $\alpha = 10^{-4}$.

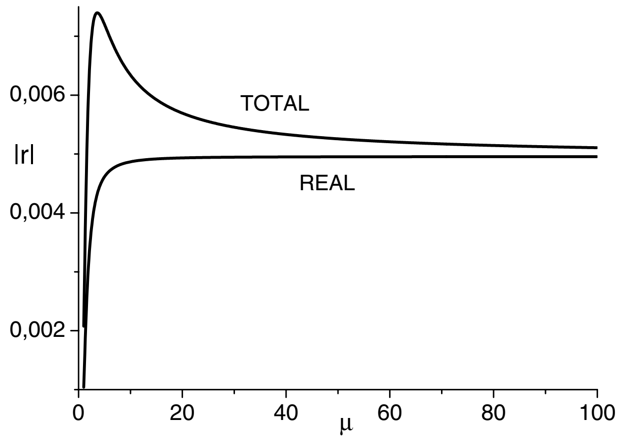

Figure 5. Absolute values of the single pulse effective 'reflection coefficient' $|r|$, calculated by means of the real part of relative frequency shift $\mathrm{Re}\,\chi(\tau)$ and the total complex function $\chi(\tau)$, as functions of parameter μ, for fixed values of parameters $B_0 = 5 \times 10^6$, $\beta = 0.1$, $\eta = 1$, $\Delta = 0.01$, and $\alpha = 10^{-4}$.

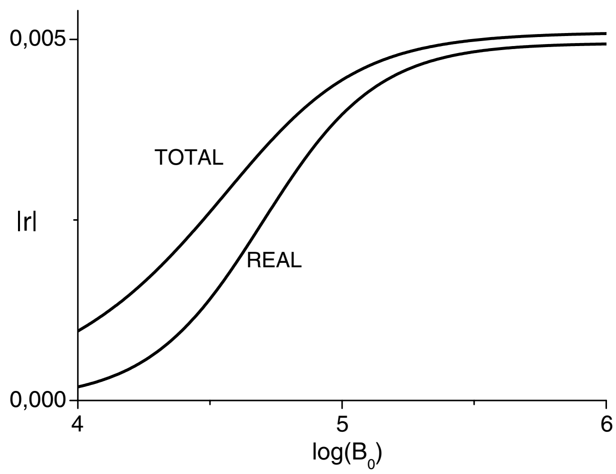

Figure 6. Absolute values of the single pulse effective 'reflection coefficient' $|r|$, calculated by means of the real part of relative frequency shift $\mathrm{Re}\,\chi(\tau)$ and the total complex function $\chi(\tau)$, as functions of parameter B_0, for fixed values of parameters $\mu = 100$, $\beta = 0.1$, $\eta = 1$, $\Delta = 0.01$, and $\alpha = 10^{-4}$.

compared with the contribution of the real part of $\chi(\tau)$, because $|\mathrm{Im}\,\chi(\tau)| \ll \mathrm{Re}\,\chi(\tau)$ in this time interval.

To estimate a possible contribution of the imaginary part of function $\chi(\tau)$ to the effective single pulse 'reflection coefficient' $|r|$, we calculated the integral (16), using the total complex function $\chi(\tau)$, given by equations (35) and (39), and only its real part. When the results are close, one may suppose that the contribution of $\mathrm{Im}\,\chi(\tau)$ can be neglected, and the theory based on the unitary evolution can be applied. Taking fixed values of coefficients $\Delta = 0.01$ and $\alpha = 10^{-4}$ and varying other parameters, we found admissible regions of parameters, where the values of $|r|$, calculated with two functions, are close.

Figure 4 shows that this happens, if β is less than 0.5, i.e., $T_r > 100$ ps. (Note that for $\beta > 1$, the value of $|r|$, calculated with the aid of the total complex function $\chi(\tau)$, declines from the constant value given by the straight line in the figure, and the two curves are very different in this domain.)

Figure 5 shows that the results become close, if $\mu > 50$, i.e., $T_c < 1$ ps. (If $\mu < 1$, the ratio of values, given by the two curves, exceeds the factor 2.)

The dependence of results on parameter B_0 is shown in figure 6. We see that the results become close for $B_0 > 5\times10^5$, while they differ by more than five times, if $B_0 < 10^4$.

By varying parameters B_0 and α simultaneously, we have verified that the crucial parameter is, indeed, the product $B_0\alpha$, and the results become close, if parameter $\mathcal{F}$ (37) exceeds the critical value $\mathcal{F}_c = 0.5$ (at least for $\mu \geqslant 100$ and $\beta \leqslant 0.1$).

5. Discussion

Hence, we have demonstrated that, indeed, it is possible to find such domains of parameters, characterizing the properties of semiconductor material (absorption coefficient, recombination time, etc) and laser pulse (intensity and time duration), where neglecting the influence of the time-dependent imaginary part of the resonance frequency shift on the photon production rate can be justified (at least, in the first approximation), so that the problem of photon creation in the selected resonance mode can be treated in the frameworks of existing unitary models. This can happen, if the crucial parameter $\mathcal{F}$ (38) exceeds the value 0.5–1, which means that the energy of laser pulses, creating electron–hole plasma, must be big enough. In order to diminish this energy, one needs materials with high mobility of carriers (the ratio τ_c/m_{ef}) and high photoabsorption efficiency. The duration of the laser pulse must be less than 1 ps.

For intensities of laser pulses exceeding the critical value (see the estimations after equation (38)), the single pulse amplitude effective 'reflection coefficient' becomes close to the asymptotical value $|r|_{max} = \eta^3\Delta/2$, corresponding to the Fresnel limit (11). Taking the relative thickness of the slab $\Delta = 0.01$, the longitudinal dimension of the cavity 11 cm and $\lambda = 12$ cm (for $f = 2.5$ GHz), one obtains for the TE_{101} mode of a rectangular cavity the value $\eta = 0.55$, so that $\eta^3 \approx 0.16$. Then, according to equation (14), one can generate $\mathcal{N} = \sinh^2(10) \sim 10^8$ photons after 12 000 pulses. If the recombination time $T_r = 600$ ps ($\beta = 0.1$), then the periodicity of laser pulses (with duration less than 1 ps) can be of the order of $T \sim (2–3)T_0 \sim 1$ ns, and the total duration of the generation process 12 μs. The necessary quality factor of the cavity is thus $Q \sim 3 \times 10^4$. Note that the recombination time does not influence the single pulse 'reflection coefficient' (provided that it is not too short; see figure 4), i.e., the total number of pulses, but it influences the interval between pulses (which cannot be less than T_r) and, consequently, the total duration of the process and the necessary Q-factor. One should try to reduce T_r to the interval 0.1–1 ns (shorter recombination times would result in diminishing the value $|r|$ and inapplicability of the quantization methods used above).

Some limitations on the energy of the laser pulse and absorption coefficient in the semiconductor slab can be derived as follows. The energy cannot be very big, in order to prevent strong heating of the material. Indeed, after the recombination time all laser energy will be released again inside a thin layer of width $l \sim \gamma^{-1}$ in the form of the energy of vibrations of atoms. It cannot be removed from the slab immediately, because the velocity of phonons is limited by the sound velocity v_s. For $D = 0.6$ mm and $v_s = 6 \times 10^5$ cm s^{-1}, the time necessary for cooling is not less than $D/v_s \sim 10^{-7}$ s. It is not reasonable to repeat laser pulses with such a great interval, because it would require a much greater quality factor of the cavity. On the other hand, many properties of semiconductors (such as mobility, for example) strongly depend on temperature. Therefore, it is desirable to avoid a big increase of temperature after each pulse, in order that the properties of the material will not change from one pulse to another. Assuming that initially the slab is at the helium temperature $T = 4$ K, one may require that the increase of temperature after each pulse will not exceed, say, $\Delta T = 0.04$ K (so that one might hope that the total increase of temperature would not exceed 4 K during the 'cooling time' of the order of 10^{-7} s). Using the Debye model, we arrive at the maximal admissible pulse energy

$$W_{\max} \approx \frac{12}{5}\pi^4 N_a S l k_B \left(\frac{T}{\Theta}\right)^3 \Delta T,$$

where N_a is the atomic concentration, k_B Boltzmann's constant, and Θ the Debye temperature. Taking $\Theta = 340$ K (as for GaAs), $N_a \sim 5 \times 10^{22}$ cm^{-3}, $S = 7 \times 2$ cm^2 and $l \sim 10^{-3}$ cm, we obtain $W_{\max} \sim 2 \times 10^{-7}$ J, which is 50 times less than another estimation after equation (38). Consequently, one needs materials with the average mobility $b > 10^7$ cm^2 V^{-1} s^{-1}. Another possibility is to use materials with smaller absorption coefficients. For instance, taking $\gamma \sim 10^2$ cm^{-1} we obtain $W_{\max} \sim 10^{-4}$ J.

Note that formula (1) for the number of generated photons in the case of a harmonically oscillating boundary can be identified with (14), if one uses the total number of oscillations instead of time and puts $\nu_{osc} \approx |r|_{osc} = (\pi/4)\Delta\eta^3$ (obviously, $\zeta = \eta^3$, and one should take into account that the maximal displacement of the boundary in this case is $2a$, where a is the amplitude of oscillations). Consequently, the 'efficiency' of sharp unharmonical changes of resonance frequency is about $2/\pi \approx 60\%$, compared with harmonical oscillations. Hence, one needs twice as many cycles as in the harmonical case, in order to generate the same number of photons for the given maximal displacement of the effective boundary. An equivalent law of motion of the boundary is given by the relation $L(t) = L_0[1 - \mathrm{Re}\,\chi(t)/\eta^2]$. The maximal velocity of the effective boundary for $D \sim 1$ mm and $T_c \sim 1$ ps is of the order of $D/T_c \sim 10^{11}$ cm s^{-1}, but since it is not related to any real material displacement, no violation of the principles of special relativity happens.

The geometrical factor η^3 in formula (35) can significantly diminish the value of the single pulse effective 'reflection coefficient' $|r|$, which would require more pulses and, consequently, more time and higher Q-factors. It was shown in [49], that in the case of a moving ideal boundary, performing *harmonical oscillations*, the use of TM modes of the field, instead of TE ones, could give certain advantages. In the appendix we generalize this result to the case of an arbitrary motion of boundary. Similar results were obtained in [47], in the case of a *spatially homogeneous* slab with harmonically oscillating in time real dielectric permeability. For an inhomogeneous slab, the shift of resonance frequency for TM modes can be found from the equation, which is a consequence of vector equation (21) for the longitudinal component $D_x = \tilde{\psi}(x)\Phi(\mathbf{r}_\perp)$ of vector $\mathbf{D}$ (this component is different from zero in the case of TM polarization) and equation (23):

$$\tilde{\psi}'' + \left[(\omega/c)^2\varepsilon(x) - k_\perp^2\right]\tilde{\psi} - \tilde{\psi}'[\varepsilon'(x)/\varepsilon(x)] = 0. \qquad (41)$$

However, this equation is more complicated than (24) for the function $\varepsilon(x) = \varepsilon_1 + \mathrm{i}B\exp(-\gamma x)$ (and the boundary conditions are also different). For this reason, we leave the problem of TM polarization in inhomogeneous slabs for a separate study.

Now, we would like to draw attention to a dangerous 'underwater rock'. The mentioned number of 10^8 photons after 12 000 pulses can be obtained, provided these pulses are strictly periodical, with very small deviations from periodicity. The problem is, that in contradistinction to the case of a harmonically oscillating boundary, when the period of wall oscillations is known in advance (it must be exactly one half of the period of electromagnetic oscillations in the selected mode), in the new scheme this period strongly depends on the phase φ of the effective inverse 'transmission coefficient', according to equation (13). But this phase turns out to be rather sensitive to the properties of material in the slab: namely, the recombination time and absorption coefficient; see figure 7 (the dependences on the parameters of pulse, μ and B_0, seem to be rather weak). It behaves approximately as $\varphi \sim \Phi(\alpha)/\beta$, where $\Phi(\alpha)$ is an increasing function of its argument. Without exact knowledge of φ, one cannot choose correctly the period of pulses (remember that a relative error in this choice cannot exceed $|r|$, otherwise there will be no constructive interference and no photon generation). This phase can be calculated, if one knows exactly the form of function $\varepsilon(x, t)$ inside the slab. The question is, however, how to reconstruct this function from experimental data. A possible solution could be to work with such materials, which would give us the values $\varphi/\pi \ll |r|$, when the correction in formula (13) could be insignificant. Unfortunately, figure 7 shows that such a situation could happen only for quite unrealistic values of β and α. (Moreover, for the values $\beta \gg 1$, one not only leaves the domain of applicability of the unitary approach, but the quantity $|r|$ also diminishes.)

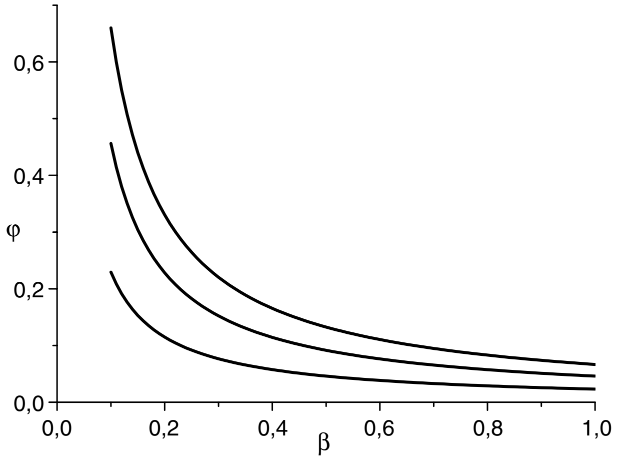

Figure 7. The phase of the single pulse inverse 'transmission coefficient' (17), calculated by means of the real part of the relative frequency shift Re $\chi(\tau)$, as a function of parameter β, for fixed values of parameters $\mu = 100$, $B_0 = 5 \times 10^6$, $\eta = 1$, $\Delta = 0.01$, and three values of parameter α: 10^{-3} (upper curve), 10^{-4} (middle curve) and 10^{-5} (lower curve).

The figures of section 4 show that the accuracy of the unitary approach (given in section 2), when applied to the calculation of the single pulse 'reflection coefficient' $|r|$ in real experimental situations, cannot be better than a few per cent in the best case. For example, figure 4 shows that, even asymptotically, the relative difference between the two values of $|r|$, obtained with the aid of the total complex function χ and its real part, remains at the level of about 10% (although the asymptotical relative differences in figures 5 and 6 are smaller). Due to the exponential dependence of the number of created photons on the number of pulses ($\mathcal{N}_n \sim \exp(2|r|n)$ for $|r|n > 1$), the relative error in prediction of the value $\mathcal{N}_n$ also grows with n as $\delta(\mathcal{N}_n)/\mathcal{N}_n \sim \exp[2\delta(|r|)n] - 1$. This means, for example, that when calculations, made in accordance with the unitary methods, give the number $\mathcal{N} \sim 10^8$ photons, the real number can differ from this quantity by almost one order of magnitude. Consequently, the development of methods of field quantization in cavities with time-dependent *complex* dielectric function (finite conductivity, nonunitary evolution) is an urgent task.

Different schemes of field quantization in stationary dissipative media were considered by many authors [59–62]. Perhaps, some of them could be generalized to the nonstationary case. Our evaluations can be improved if one considers more realistic dependences of the complex dielectric function on space and time coordinates inside the semiconductor slab. In particular, using formula (2) we did not take into account the effects of dispersion, which are known to be very important for the stationary Casimir effect [63]. The choice of function (18) was made mainly because of its simplicity and reasonable behaviour for large values of the coordinate. However, the results of studies [64–66] show that the concentration of carriers created inside the slab due to photoabsorption must have zero derivative at $x = 0$, so that a more realistic function could have the form $\varepsilon_2(x) = B/\cosh(\gamma x)$ (or some combinations of powers of this function). The function (39) also should be considered only as the simplest model. More adequate functions $\varepsilon_2(x, t)$ (which are not factorized with respect to the variables x and t for an arbitrary shape of the laser pulse) can be calculated using the approaches developed in [64–66].

However, the study of the simple model of section 3 is very important, because having analytical results one can estimate the accuracy of numerical calculations and different approximations. In particular, comparing numerical solutions of equation (31) with formula (33), we have verified that the relative difference between numerical and analytical results in this case is less than 2×10^{-3}. This means that solving equation (24) with an arbitrary function $\varepsilon(x)$ in the region $0 < x < D$ one can replace the wavenumber k (defined in equation (26)) by its unperturbed value π/L. Then the correction ξ can be calculated as $\xi = \psi(0)/[L\psi'(0)]$, where $\psi(x)$ is the solution of equation (24) with $k = \pi/L$ for $0 < x < D$, satisfying the boundary condition $\psi(D) = 0$, and $\psi'(x)$ is the derivative of this function.

Acknowledgments

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Appendix. Comparison of effective reflection coefficients for TE and TM modes in cavities with moving boundaries

The function $\omega(t)$ depends on the geometry and *polarization* of the field inside the 3D cavity. In certain simple cases (e.g., for a cylindrical geometry), when the field can be decomposed into TE or TM modes, the set of Maxwell equations can be reduced to a scalar wave equation for some function, which determines the electric and magnetic fields (here we put $c = 1$, and the cavity is supposed to be empty): $\psi_{tt} - \psi_{xx} - \Delta_\perp \psi = 0$ (x is the direction along which the motion of a flat boundary takes place). For TE modes one has Dirichlet boundary conditions $\psi(0; y, z; t) = \psi(L(t); y, z; t) = 0$, which allow us to factorize the function $\psi(x; y, z; t)$ as

$$\psi(x; y, z; t) = q(t)L^{-1/2}(t)\sin[\pi x/L(t)]\Phi(y, z),$$

where function $\Phi(y, z)$ obeys the two-dimensional Helmholtz equation (23). Then the time-dependent 'field amplitude' $q(t)$ satisfies the harmonic oscillator equation (7) with $\omega_{\mathrm{TE}}^2 = k_\perp^2 + \pi^2/L^2(t)$.

TM modes must obey the generalized Neumann condition [67] (which means that the four-dimensional gradient of ψ must be orthogonal to the world line of the mirror in the Minkowsky space) $(\partial_x + \dot{L}(t)\partial_t)\psi(L(t); y, z; t) = 0$. Following the recipe of [49], we choose the function ψ in the form

$$\psi(x; y, z; t) = \{q(t) + \dot{q}(t)\dot{L}L(t)v[x/L(t)]\} \\ \times L^{-1/2}(t)\cos[\pi x/L(t)]\Phi(y, z),$$

where an auxiliary function $v(z)$ satisfies the conditions $v(0) = v(1) = v'(0) = 0$ and $v'(1) = -1$. In this case, the equation for $q(t)$ is more complicated:

$$\ddot{q} + \omega_{\rm TM}^2(t) q = \mathcal{A}\left\{q''' L\dot{L} + 2\ddot{q} L\ddot{L} + \dot{q}\left(LL''' + \pi^2 \dot{L}/L\right)\right\},$$

$$\mathcal{A} = 2\int_0^1 \cos^2(\pi z) v(z)\, {\rm d}z,$$

but one can nullify the coefficient $\mathcal{A}$, choosing in a proper way the auxiliary function $v(z)$. The time-dependent frequency in this case is determined by the formula

$$\omega_{\rm TM}^2 = k_\perp^2 + \pi^2/L^2(t) - \ddot{L}/L(t).$$

If $L(t) = L_0[1 + a(t)]$ with $|a(t)| \ll 1$, then, using equation (16), one can verify that, in both cases, the effective reflection coefficients are proportional to the Fourier transform of function $a(t)$, but with different coefficients:

$$|r| = G\left|\int_{-\infty}^{\infty} \omega_0 a(t) {\rm e}^{-2{\rm i}\omega_0 t}\, {\rm d}t\right|, \qquad \text{(A.1)}$$

$$G_{\rm TE} = 1 - \frac{k_\perp^2}{\omega_0^2} \equiv \eta^2, \qquad G_{\rm TM} = 1 + \frac{k_\perp^2}{\omega_0^2} \equiv 2 - \eta^2, \quad \text{(A.2)}$$

where the coefficient $\eta \equiv \lambda/2L_0$ is the same as in the main text. For harmonical oscillations of the boundary, an equivalent result was obtained in [49]. We see that modes with TM polarization have some advantages over TE modes, from the point of view of giving greater values of $|r|$, especially if η is essentially smaller than 1. However, one should not think that $r_{\rm TM}$ would remain finite even for very long cavities with $\eta \ll 1$. If the *dimensional* amplitude of oscillations A of the boundary is fixed, as well as the resonance frequency, then $a = (2A/\lambda)(\lambda/2L_0)$, so that one should analyse the behaviour of the renormalized coefficient $\tilde{G}_{\rm TM} = \eta G_{\rm TM} = \eta(2 - \eta^2)$. It attains the maximal value $\tilde{G}_{\rm TM}^{(\max)} = \sqrt{32/27} \approx 1.1$ for $\eta_* = \sqrt{2/3} \approx 0.8$ (when the coefficient $\tilde{G}_{\rm TE} = \eta G_{\rm TE} = \eta^3$ equals approximately 0.5). For small values $\eta \ll 1$, the coefficient $\tilde{G}_{\rm TM}$ goes to zero as 2η, whereas $\tilde{G}_{\rm TE}$ decreases as η^3 (both for the real moving mirror and its plasma mirror simulation, as shown in the main text).

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