Continuous measurement of atom-number moments of a Bose-Einstein condensate by photodetection

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We propose a measurement scheme that allows determination of even moments of a Bose-Einstein condensate (BEC) atom number, in a ring cavity, by continuous photodetection of an off-resonant quantized optical field. A fast cavity photocounting process limits the heating of atomic samples with a relatively small number of atoms, being convenient for BECs on microchip scale applications. The measurement back-action introduces a counting-conditioned phase damping, suppressing the condensate typical collapse and revival dynamics.

The recent achievement of Bose-Einstein condensates (BECs) trapped near the surface of magnetic microchip traps [1] has led to a new promising system for the development of emerging technologies based on BECs, such as trapped-atom interferometry [2] or atom-based quantum information processing (QUIP) [3], due to the high degree of control achieved over the atomic sample. A fundamental issue for implementing those technologies on a chip scale is the achievement of a nondestructive measurement of the BEC properties. In particular, QUIP calls for high precision nondestructive detection of the BEC atom number [3], which has proven to be a hard task, attracting considerable attention [4–6].

Since the very early experiments with diluted trapped neutral atoms [7], the BEC dynamics monitoring has been achieved either by absorption or dispersive imaging [8]. Absorption imaging has the countereffect of heating up the condensate, precluding it for latter usage (destructive regime). On the other hand in dispersive imaging the small phase-shift suffered by the far-detuned probe light is compensated by a high intensity. Residual incoherent Rayleigh scattering heats up the atomic sample through spontaneous emission atomic recoil, preventing a nondestructive regime as well [8] for the reduced number of atoms in microchip BECs ($\approx 10^4$) [1]. Thus, it is certainly worthwhile to propose alternative schemes of atom detection that besides being nondestructive to some extent, could also be useful for feedback and control of the condensate—a valuable resource for QUIP.

In this Rapid Communication we investigate the information extracted about a BEC atom number through probe-field continuous photodetection. Previous treatments on BEC continuous measurements have been described in Refs. [9–12], differing considerably from our approach and goals. We consider a BEC trapped inside a ring cavity fed by two resonant (orthogonally polarized) propagating fields—an undepleted probe and a weak quantum probe field (Fig. 1). The presence of the undepleted pump field allows that the moments of the

detected probe field photon number give direct information about even moments of the BEC atom number. Moreover, since the condensate atom number information is carried by the probe field photocounting statistics, there is no need for a strong probe field, avoiding thus heating during the measurement process. Finally, we discuss how the detection back action induces phase uncertainty to the condensate state, suppressing its original collapse and revival dynamics.

The system, depicted in Fig. 1, consists of a Schrödinger field of bosonic two-level atoms with transition frequency ν_0 interacting via electric dipole with the two single-mode orthogonally polarized ring-cavity probe and pump fields of frequencies ν_1 and ν_2, respectively, both being far-off resonant from any electronic transition (calculation details given in Ref. [13]). The eigenstates for the atoms are denoted by $|k\rangle$ with eigenfrequencies ω_k, whose values are dependent on the trapping conditions. For an atomic cloud well localized both longitudinally and transversally relative to the cavity roundtrip (L) and to the cavity field beam waist (S), respectively, the field can be assumed uniform in its vicinity, such that the coupling between atoms and pump and probe fields is approximately constant. In the far-off resonance regime the k-excited state population is negligible, and the collision
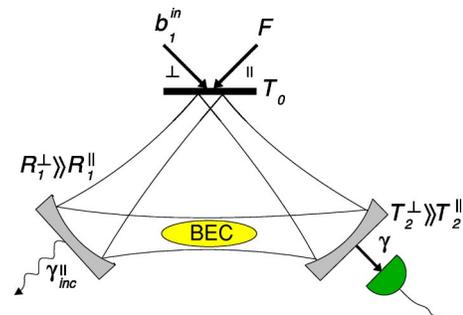

FIG. 1. BEC in a ring cavity setup. The pump (F) and probe ($b_1^i n$) input fields are ∥ and ⊥ polarized, respectively. Mirror 1 and 2 reflectivities are polarization selective, in order that the in-cavity pump probe is heavily damped at mirror 1, while the transmissivity at mirror 2 allows that BEC properties be determined by the probe field photocounting at the mirror 2 output.

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between excited atoms, and between the excited and the ground state atoms can be neglected. In a rotating referential with the frequency ν_2 the Hamiltonian shows

$$H = \hbar \sum_k \left[\omega_k c_k^\dagger c_k + \Delta_k a_k^\dagger a_k \right] + \hbar \sum_{jklm} \kappa_{jklm} c_j^\dagger c_k^\dagger c_l c_m$$
$$+ \hbar \sum_{n=1}^{2} \sum_{jk} (g_n \langle j | e^{i\mathbf{k}_n \cdot \mathbf{r}} | k \rangle b_n a_j^\dagger c_k + \text{H.c.})$$
$$+ \hbar \delta b_1^\dagger b_1 + \hbar (F b_2^\dagger + F^* b_2), \tag{1}$$

where $\Delta_k = \omega_k + \Delta$ and $\delta = \nu_2 - \nu_1$, being $\Delta = \nu_0 - \nu_2$ is the detuning between pump and atom. c_k and a_k are the annihilation operators for atoms with k in the ground and excited state, respectively, and κ_{jklm} is the collision strength between ground state atoms. The third term on the right-hand side of Eq. (1) is the interaction between the atoms and the probe (b_1) and pump (b_2) fields (with coupling constants g_1 and g_2, respectively), whose wave vectors $\mathbf{k}_{1(2)}$ must satisfy $|\mathbf{k}_{1(2)}| = 2\pi n/L$, with n integer. The in-cavity probe field is related to the input field (b_1^{in}) by $b_1 = \sqrt{T_0} b_1^{in}$ (neglecting fluctuations), where T_0 is the mirror 0 transmission index. The field b_2 external pumping is given by the last term of Eq. (1), where $|F|$ is the external resonant driving field strength. If the pump beam cavity loss is considerably higher than the coupling constants the pump average photon number can be kept constant (undepleted), due to the pump-loss competition. This assumption allows the pump field to be treated as a c number, and also avoids the atomic sample heating through residual incoherent Rayleigh scattering by setting a low steady pump intensity. Since we also require that the probe field loss rate is smaller than the photocounting rate, the pump is set to a $\parallel$-polarization (to the table top) while the probe is set to a $\perp$-polarization. The cavity mirror 1 thus must have distinct reflection indexes $R_1^\perp \gg R_1^\parallel$. Assuming the bad-cavity limit for the $\parallel$-polarization ($\gamma_{\text{inc}}^\parallel \gg |g_1|^2/\gamma_{\text{inc}}^\parallel$, $|g_2|^2/\gamma_{\text{inc}}^\parallel \gg \Gamma$), with $\gamma_{\text{inc}}^\parallel \propto T_1^\parallel = 1 - R_1^\parallel$ and Γ the atomic spontaneous emission rate, the pump field can be adiabatically eliminated such that b_2 can be replaced by $-iF/\gamma_{\text{inc}}^\parallel$. Remark that the probability of atomic spontaneous emission (P_e) is also reduced inside resonators [4,14] with high finesse $\mathcal{F}$, since the *per photon* probability of spontaneous emission goes with $P_e \propto \mathcal{F}$, and the required number of the probe beam photons for reliable detection is $\bar{N} \propto \mathcal{F}^{-2}$, thus the total number of spontaneous scattering events is $\bar{N} P_e \propto \mathcal{F}^{-14}$.

In the limit of large detuning $|g_i/\Delta| \ll 1$, $i = 1, 2$, and $\omega_k/\Delta \ll 1$, $\Delta_k \approx \Delta$ [15]. Thus, atomic spontaneous emission can be neglected and the excited states operators a_k are eliminated adiabatically resulting in the following effective Hamiltonian:

$$H_{\text{eff}} = \hbar \delta b^\dagger b + \hbar \sum_k \left(\omega_k + \frac{|\widetilde{g_2}|^2}{\Delta} \right) c_k^\dagger c_k + \hbar \sum_{klmn} \kappa_{klmn} c_k^\dagger c_l^\dagger c_m c_n$$
$$+ \hbar \frac{|g_1|^2}{\Delta} b^\dagger b \sum_k c_k^\dagger c_k + \hbar \sum_{kl} \left(\frac{g_1^* \widetilde{g_2}}{\Delta} \langle k | e^{-i(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r}} | l \rangle b^\dagger \right.$$
$$+ \left. \frac{g_1 \widetilde{g_2}^*}{\Delta} \langle k | e^{i(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r}} | l \rangle b \right) c_k^\dagger c_l, \tag{2}$$

where $\widetilde{g_2} \equiv -i g_2 F / \gamma_{\text{inc}}^\parallel$ is the effective coupling, and we have defined $b_1 \equiv b$. Hamiltonian (2) is the prototype for atom-optic parametric amplification [16], where atoms in the ground state are transferred to side mode states. However, we are interested in the situation where no optical intermode excitation occurs. In the ring-cavity arrangement $\mathbf{k}_{1(2)}$ (with $|\mathbf{k}_{1(2)}| = 2\pi n/L$) are both colinear to the longitudinal dimension of the condensate L_c, which is taken to be very small compared to the cavity roundtrip length L. Thus $\langle k | e^{i(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r}} | l \rangle \approx \delta_{k,l}$ whenever $2\pi n L_c / L \rightarrow 0$, and no intermode excitation occurs. This embodies the specific case of $\mathbf{k}_1 \approx \mathbf{k}_2$ (and thus $\delta = 0$), which we consider hereafter. To simplify we further assume a pure condensate with all atoms in the c_0 mode, the Hamiltonian finally reduces to

$$H_{\text{eff}} = \hbar \left(\omega_0 + \frac{|\widetilde{g_2}|^2}{\Delta} \right) c_0^\dagger c_0 + \hbar \kappa c_0^\dagger c_0^\dagger c_0 c_0 + \hbar \frac{|g_1|^2}{\Delta} b^\dagger b c_0^\dagger c_0$$
$$+ \hbar \left(\frac{g_1^* \widetilde{g_2}}{\Delta} b^\dagger + \frac{g_1 \widetilde{g_2}^*}{\Delta} b \right) c_0^\dagger c_0. \tag{3}$$

In Eq. (3) we identify two regimes in the interplay between the pump and probe fields strength: (i) Whenever $|\widetilde{g_2}/g_1| \ll 1$ the strongest contribution is from the quantum probe field, including the situation without the classical pump field; (ii) otherwise the classical pump field has an important contribution to the effective Hamiltonian. Equation (3) shows that the condensate atom number $n_0 = c_0^\dagger c_0$ is a non-demolition variable. By varying $|F/\gamma_\parallel|$ and thus $|\widetilde{g_2}|$ distinct regimes of quantum nondemolition couplings [17–19] are attained. For $|\widetilde{g_2}/g_1| \ll 1$ the nondemolition regime corresponds to that considered in Refs. [10,11] for BECs atom number nondemolition measurement, while for $|\widetilde{g_2}/g_1| \gtrsim 1$ features similar to the photon number nondemolition measurements discussed in Ref. [20] are added.

Now we turn to the photodetection process. To simplify the photocounting modeling [21] we first assume that no other incoherent process, such as $\perp$-polarized photon losses, considerably affects the the probe field dynamics over the counting time interval. This assumes $\gamma_{inc}^\perp \ll \gamma$, where γ is the effective cavity photodetection rate given by $\gamma \approx T_2^\perp \eta$, where $T_2^\perp$ is the mirror 2 transmission coefficient and η is the output field photodetection rate, neglecting output field fluctuations [13,22]. The counting of k photons from the probe field in a time interval t can be characterized by the linear operation $N_t(k)$ [21], acting on the state of the system as $\rho^{(k)}(t) = N_t(k)\rho(0)/\text{Tr}[N_t(k)\rho(0)]$ where $\rho(0)$ is the joint state of the condensate and the probe field prior turning on the counting process, with probability $P(k,t) = \text{Tr}[N_t(k)\rho(0)]$. The operation $N_t(k)$ is written as

$$N_t(k) = \int_0^t dt_k \int_0^{t_k} dt_{k-1} \cdots \int_0^{t_2} dt_1 S_{t-t_k} J S_{t_k - t_{k-1}} \cdots J S_{t_1}, \tag{4}$$

where $S_t \rho = e^{Yt} \rho e^{Y^\dagger t}$, with $Y = -\frac{i}{\hbar} H - R/2$. H is the system Hamiltonian, and $R = \gamma b^\dagger b$ is the counting rate operator. As such $J\rho \equiv \gamma b \rho b^\dagger$ indicates the change of the probe field due

to the loss of one counted photon, while S_t is responsible for the state evolution between counts.

From Eq. (3) Y becomes

$$Y = -i\left(\omega_0 - \kappa + \frac{\tilde{g}_2^2}{\Delta}\right)n_0 - i\kappa n_0^2 - i\left(\delta_{n_0} - i\frac{\gamma}{2}\right)b^\dagger b$$
$$- i(F_{n_0}^* b + F_{n_0} b^\dagger), \tag{5}$$

where we defined $\delta_{n_0} \equiv (|g_1|^2/\Delta)n_0$, $F_{n_0} \equiv (g_1^* \tilde{g}_2/\Delta)n_0$. We express the $N_t(k)$ acting on the joint initial state $\Sigma_m C_m |m\rangle \otimes |\beta\rangle$, where the first ket stands for the condensate state while the second is the probe-field state, hereafter assumed as coherent.

After k-count events on the probe field, the conditioned joint state becomes

$$\rho^{(k)}(t) = \frac{1}{k!P(k,t)} \sum_{m,m'} C_m C_{m'}^* \mathcal{F}_{m,m'}^k(t)$$
$$\times e^{\Phi_m(t) + \Phi_{m'}^*(t)} |m\rangle\langle m'| \otimes |\beta_m(t)\rangle\langle\beta_{m'}(t)|, \tag{6}$$

where

$$\mathcal{F}_{m,m'}(t) \equiv \gamma \left\{ -\frac{\Lambda_m \Lambda_{m'}^*}{\Gamma_m + \Gamma_{m'}^*}[e^{-(\Gamma_m + \Gamma_{m'}^*)t} - 1] + G_m G_{m'}^* t \right.$$
$$\left. + i\left[\frac{G_m \Lambda_{m'}^*}{\Gamma_{m'}^*}(e^{-\Gamma_{m'}^* t} - 1) - \frac{G_{m'}^* \Lambda_m}{\Gamma_m}(e^{-\Gamma_m t} - 1)\right] \right\}, \tag{7}$$

with $\Gamma_m = (i\delta_m + \gamma/2)$, $G_m = F_m/\Gamma_m$, and $\Lambda_m = \beta + iG_m$, for $\delta_m = (|g_1|^2/\Delta)m$ and $F_m = (g_1^* \tilde{g}_2/\Delta)m$. In Eq. (6), $\beta_m(t) \equiv \Lambda_m e^{-\Gamma_m t} - iG_m$ is the label for the probe field coherent state:

$$\Phi_m(t) \equiv -\frac{1}{2}(|\beta|^2 - |\beta_m(t)|^2) + i[G_m \Lambda_m(e^{-\Gamma_m t} - 1)$$
$$+ (i|G_m|^2 \Gamma_m^* - \theta_m)t], \tag{8}$$

and $\theta_m \equiv [\omega_0 + |g_2|^2/\Delta + \kappa(m-1)]m$ is a phase introduced by the atomic collision process and the classical pump. The last two terms of $\Phi_m(t)$, Eq. (8), besides a direct collision process also include the terms $|G_m|^2$ and $G_m \Lambda_m(e^{-\Gamma_m t} - 1)$, which are originated by the pump field, inducing a collision-like behavior, with diffusion of the condensate state phase.

The probability to count k photons during the time interval t is given by

$$P(k,t) = \frac{1}{k!} \sum_m |C_m|^2 \mathcal{F}_{m,m}^k(t) e^{-\mathcal{F}_{m,m}(t)}. \tag{9}$$

In regime (i), $\tilde{g}_2/g_1 \ll 1$, the counting probability Eq. (9) reduces to the Poisson distribution

$$P(k,t) = \frac{1}{k!}[|\beta|^2(1 - e^{-\gamma t})]^k e^{-|\beta|^2(1 - e^{-\gamma t})}, \tag{10}$$

independently of the condensate state and the atom-field coupling as well. The r moments of $P(k,t)$ for this regime are

$\overline{k^r} = [|\beta|^2(1 - e^{-\gamma t})]^r$, and simply relate to the probe amplitude. However in regime (ii), Eq. (9) must be fully considered, and the condensate state is relevant for the photocounting probability distribution. Thus inference about the condensate atom number moments can be given by the photocounting distribution. The r-moments of Eq. (9) are

$$\overline{k^r} = \sum_m |C_m|^2 \mathcal{F}_{m,m}^r(t) = \langle \mathcal{F}_{n_0,n_0}^r(t)\rangle, \tag{11}$$

which in the long time limit ($\gamma t \gg 1$) goes to

$$\overline{k^r} \approx (\gamma t)^r \left\langle \left(\frac{\left|\frac{g_1}{\Delta}\right|^2 n_0^2}{\left|\frac{\gamma}{2\tilde{g}_2}\right|^2 + \left|\frac{g_1}{\Delta}\right|^2 \left|\frac{g_1}{\tilde{g}_2}\right|^2 n_0^2}\right)^r \right\rangle. \tag{12}$$

The limit $\gamma/2\Delta \gg |g_1/\Delta|^2$ gives the central result of this paper, since we may approximate Eq. (12) by

$$\overline{k^r} \approx (\gamma t)^r \left|\frac{2g_1\tilde{g}_2}{\gamma\Delta}\right|^{2r} \langle n_0^{2r}\rangle, \tag{13}$$

and the even moments of the condensate atom number are directly given by the moments of the number of photocounts. Particularly, for a BEC in a Fock state $\sqrt{k}$ gives a null uncertainty measure of the condensate $\langle n_0\rangle$.

In the opposite limit, $\gamma/2\Delta \ll |g_1/\Delta|^2$, the photocounting moments give

$$\overline{k^r} \approx (\gamma t)^r \left|\frac{\tilde{g}_2}{g_1}\right|^{2r}, \tag{14}$$

and thus the fields strength ratio is dynamically probed *in situ*, while the condensate is inside the cavity, by the determination of the average number of counted photons at the slow rate $\gamma/2\Delta \ll |g_1/\Delta|^2$.

The important time scale parameter for determination of the condensate atom number even moments by photocounting is the effective photocounting rate γ. Since the undepleted classical pump field approximation is valid only in the ∥-polarization bad-cavity limit ($\gamma_{inc}^\parallel \gg |g_1|^2/\gamma_{inc}^\parallel, |g_2|^2/\gamma_{inc}^\parallel$) we must also have $\gamma_{inc}^\parallel \gg \gamma$. The ability to build up a ring cavity with high finesse at the microchip surface could represent a restriction, but recent effort has been made in the study of properties of ultracold atomic samples inside a ring cavity, which could attain finesses as high as 170 000 [14]. In fact, a high finesse cavity is necessary only when the small phase shift has to be compensated by a large intensity field, such as in dispersive imaging, since information about the BEC is carried by the probe field phase. However, in our proposal the pump and probe intracavity fields can be both set at low intensity, which limits the effects of incoherent Rayleigh scattering through spontaneous emission during the photocounting period. If every atomic spontaneous emission heats the condensate in about an atomic recoil energy E_R, we can estimate the total heating due the fraction $N_e = P_e\langle n_0\rangle$ of atoms suffering spontaneous emission, where $P_e \propto \Gamma/\Delta^2$ is the per photon spontaneous emission probability in the far-off resonance regime with the intracavity spontaneous emis-

sion rate Γ. The BEC heating due the interaction with the probe light with $I = \langle b^\dagger b \rangle$ photons amounts to $\Delta T \propto 2E_R I \langle n_0 \rangle \Gamma / 3k_B \Delta^2$. For the regime of $\gamma / 2\Delta \gg |g_1 / \Delta|^2$ of optimal detection of the atom number moments we can set a limiting Γ such that the heating is negligible, i.e., by considering $\Gamma \ll |g_1|^2 / \gamma_{\text{inc}}^{\parallel}, |g_2|^2 / \gamma_{\text{inc}}^{\parallel} \ll \gamma \ll \gamma_{\text{inc}}^{\perp}$. Since $\gamma = \eta T_2^{\perp}$, the above limit can be conveniently reached with a high $\perp$-transmission coefficient mirror and a reasonably fast photodetector.

Despite the heating process being negligible there will always be a backaction on the condensate state due the continuous measurement process. Only if the condensate is initially in a Fock state, an eigenstate of the nondemolition variable, is that the condensate will evolve freely independently of the counting probability. The same is valid for the diagonal elements of the unconditioned state:

$$\rho_c(t) = \sum_k P(k,t) \rho_c^{(k)}(t)$$
$$= \sum_{m,m'} C_m C_{m'}^* e^{\Phi_m(t) + \Phi_{m'}^*(t) + \mathcal{F}_{m,m'}(t)}$$
$$\times \langle \beta_{m'}(t) | \beta_m(t) \rangle |m\rangle\langle m'|, \qquad (15)$$

since $\langle m | \rho_c(t) | m \rangle = |C_m|^2$. The off-diagonal elements of Eq. (15) are evidence of back-action over the condensate state phase. Obviously, this implies that a condensate in a completely mixed state will not suffer the back-action effects. Any other condensate state will be affected by the collision-like terms $|G_m|^2$ and $G_m \Lambda_m (e^{-\Gamma_m t} - 1)$ from Eq. (8). The counting process induces an irreversible phase damping, inhibiting the well known coherent collapse and revival dynamics of the condensate state [23]. The k-counts condi-

tioned phase damping does not appear when no photons are detected, $k=0$, and the BEC state evolves with its typical collapse and revival dynamics.

In conclusion, we have investigated the measurement over the BEC inside a ring cavity that can be achieved through continuous photodetection of a quantum probe field. Even-moments of the condensate atom number can be inferred by the probe field photodetection probability distribution whenever the photodetector counting rate follows $\gamma / 2\Delta \gg |g_1 \Delta|^2$. Also, if those rates are higher than the atomic spontaneous emission rate the condensate heating will be prevented. Although the atom number is a QND variable, there is a back action on the condensate state due to the counting process, inducing phase damping over the condensate state whenever photons are counted. The strong dependence of the photocounting probability distribution with the BEC original state suggests that this measurement scheme can be a useful resource for feedback and control of atomic samples. Further investigation on those issues for monitoring of cross-correlation between atoms and light fields together with calculations on signal to noise ratio, as well as a measurement resource for atom based quantum information processing will be addressed elsewhere [13].

It is still unknown whether surface interactions reinforced by the cavity will introduce noise limiting the detection process. Besides technical problems yet to be solved for cavity quantum electrodynamics implementation on microchips [4], we believe that the above proposal could be implemented, in principle, due to the rapid advance on experimental research.

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